EXTREME PRECIPITATION TRENDS OVER THE CONTINENTAL UNITED STATES

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STATEMENT OF PROBLEM

During the past decade, there has been extensive research by climatologists documenting increases in the levels of extreme precipitation, both in observational and model-generated data. Groisman *et al.* (*Journal of Climate*, 2005) have a comprehensive review of this whole field.

With a few exceptions (papers by Katz, Zwiers and co-authors) this literature have not made use of the extreme value distributions and related constructs

There are however a few papers by statisticians that have explored the possibility of using more advanced extreme value methods (e.g. Cooley, Naveau and Nychka, *JASA* 2007; Sang and Gelfand, *Env. Ecol. Stat.* 2008)

In this paper, we explore systematically the development of extreme value and spatial models for this problem

CURRENT METHODOLOGY

(see e.g. Groisman *et al.* 2005)

Most common method is based on counting exceedances over a high threshold (e.g. 99.7% threshold)

- Express exceedance counts as anomalies from 30-year mean at each station
- Average over regions using "geometric weighting rule": first average within 1^{o} grid boxes, then average grid boxes within region
- Calculate standard error of this procedure using exponential spatial covariances with nugget (range of 30–500 km, nugget-sill ratio of 0–0.7).
- Increasing trends found in many parts of the world, nearly always stronger than trends in precipitation means, but spatially and temporally heterogeneous. Strongest increase in US extreme precipitations is post-1970, about 7% overall

DATA SOURCES

- NCDC Rain Gauge Data (Groisman 2000)
  - Daily precipitation from 5873 stations
  - Select 1970–1999 as period of study
  - 90% data coverage provision — 4939 stations meet that

- NCAR-CCSM climate model runs
  - 20×41 grid cells of side 1.4^{o}
  - 1970–1999 and 2070–2099 (A2 scenario)

- PRISM data
  - 1405×621 grid, side 4km
  - Elevations
  - Mean annual precipitation 1970–1997

EXTREME VALUE DISTRIBUTIONS

Suppose $X_1, X_2, ...,$ are independent random variables with the same probability distribution, and let $M_n = \max(X_1, ..., X_n)$. Under certain circumstances, it can be shown that there exist *normalizing constants* $a_n > 0, b_n$ such that

$$\Pr\left\{\frac{M_n - b_n}{a_n} \leq x\right\} = F(a_n x + b_n)^n \to H(x).$$

The *Three Types Theorem* (Fisher-Tippett, Gnedenko) asserts that if nondegenerate H exists, it must be one of three types:

$$H(x) = \exp(-e^{-x}), \text{ all } x \quad \text{(Gumbel)}$$

$$H(x) = \begin{cases} 0 & x < 0 \\ \exp(-x^{-\alpha}) & x > 0 \end{cases} \quad \text{(Fréchet)}$$

$$H(x) = \begin{cases} \exp(-|x|^{\alpha}) & x < 0 \\ 1 & x > 0 \end{cases} \quad \text{(Weibull)}$$

In Fréchet and Weibull, $\alpha > 0$.

The three types may be combined into a single *generalized extreme value* (GEV) distribution:

$$H(x) = \exp\left\{-\left(1+\xi\frac{x-\mu}{\psi}\right)_+^{-1/\xi}\right\},$$

$(y_+ = \max(y, 0))$

where μ is a location parameter, $\psi > 0$ is a scale parameter and ξ is a shape parameter. $\xi \to 0$ corresponds to the Gumbel distribution, $\xi > 0$ to the Fréchet distribution with $\alpha = 1/\xi$, $\xi < 0$ to the Weibull distribution with $\alpha = -1/\xi$.

$\xi > 0$: "long-tailed" case, $1 - F(x) \propto x^{-1/\xi}$,

$\xi = 0$: "exponential tail"

$\xi < 0$: "short-tailed" case, finite endpoint at $\mu - \xi/\psi$

EXCEEDANCES OVER THRESHOLDS

Consider the distribution of X conditionally on exceeding some high threshold u:

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)}.$$

As $u \to \omega_F = \sup\{x : \ F(x) < 1\}$, often find a limit

$$F_u(y) \approx G(y; \sigma_u, \xi)$$

where G is *generalized Pareto distribution* (GPD)

$$G(y; \sigma, \xi) = 1 - \left(1 + \xi \frac{y}{\sigma}\right)_+^{-1/\xi}.$$

The Generalized Pareto Distribution

$$G(y;\sigma,\xi) = 1 - \left(1 + \xi\frac{y}{\sigma}\right)_+^{-1/\xi}.$$

$\xi > 0$: long-tailed (equivalent to usual Pareto distribution), tail like $x^{-1/\xi}$,

$\xi = 0$: take limit as $\xi \to 0$ to get

$$G(y;\sigma,0) = 1 - \exp\left(-\frac{y}{\sigma}\right),$$

i.e. exponential distribution with mean σ,

$\xi < 0$: finite upper endpoint at $-\sigma/\xi$.

The *Poisson-GPD model* combines the GPD for the excesses over the threshold with a Poisson distribtion for the number of exceedances. Usually the mean of the Poisson distribution is taken to be λ per unit time.

POINT PROCESS APPROACH

Homogeneous case:

Exceedance $y > u$ at time t has probability

$$\frac{1}{\psi}\left(1+\xi\frac{y-\mu}{\psi}\right)_+^{-1/\xi-1}\exp\left\{-\left(1+\xi\frac{u-\mu}{\psi}\right)_+^{-1/\xi}\right\}dydt$$

- $\mu,\ \psi,\ \xi$ are GEV parameters for annual maxima
- N-year return value — the level y_N that is exceeded in any one year with probability $\frac{1}{N}$.

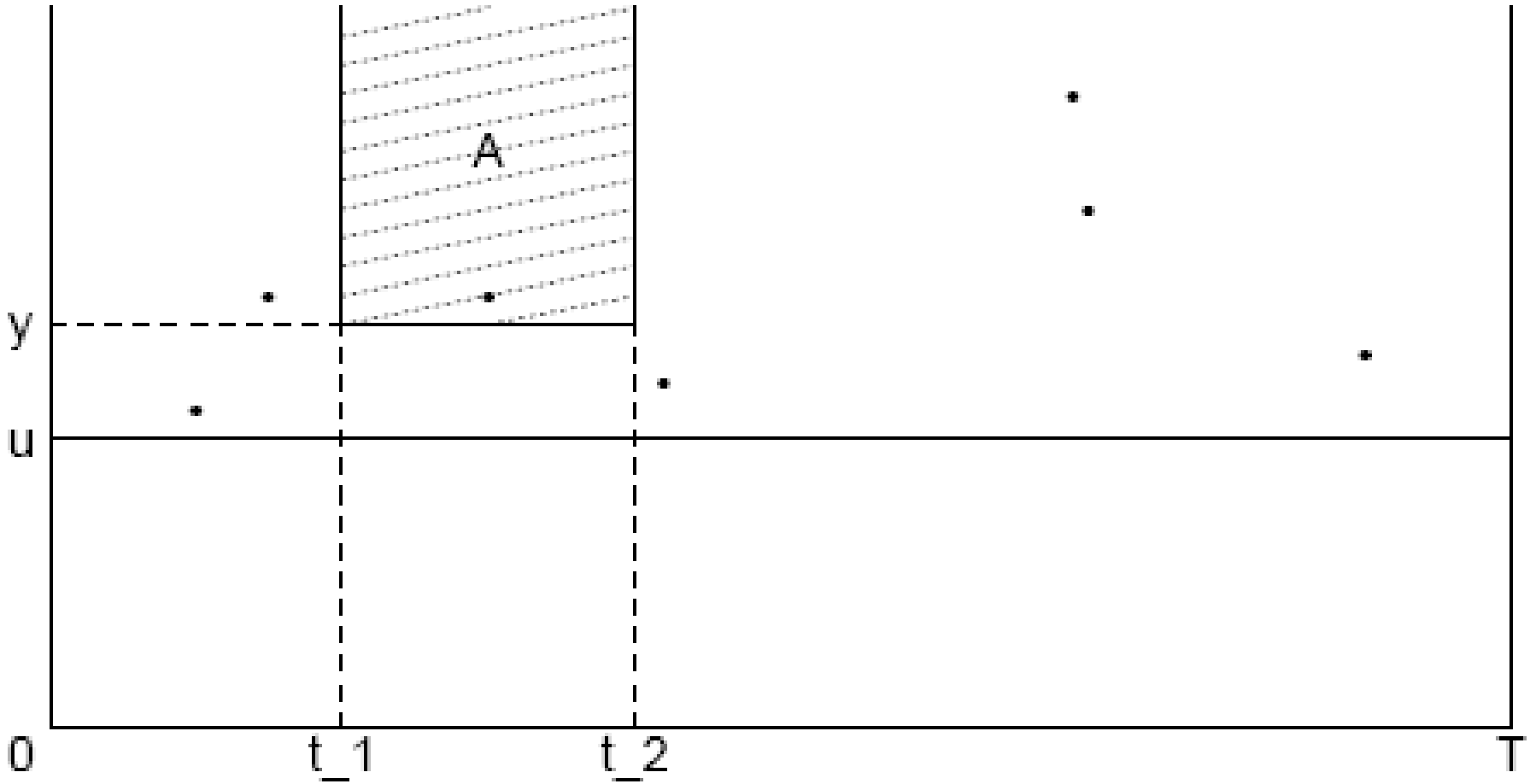


Illustration of point process model.

Inhomogeneous case:

- Time-dependent threshold u_t and parameters $\mu_t,\ \psi_t,\ \xi_t$
- Exceedance $y > u_t$ at time t has probability

$$\frac{1}{\psi_t}\left(1+\xi_t\frac{y-\mu_t}{\psi_t}\right)_+^{-1/\xi_t-1}\exp\left\{-\left(1+\xi_t\frac{u_t-\mu_t}{\psi_t}\right)_+^{-1/\xi_t}\right\}dydt$$

- Estimation by maximum likelihood

Seasonal models without trends

General structure:

$$\begin{aligned}
\mu_t &= \theta_{1,1} + \sum_{k=1}^{K_1} \left(\theta_{1,2k} \cos \frac{2\pi kt}{365.25} + \theta_{1,2k+1} \sin \frac{2\pi kt}{365.25} \right), \\
\log \psi_t &= \theta_{2,1} + \sum_{k=1}^{K_2} \left(\theta_{2,2k} \cos \frac{2\pi kt}{365.25} + \theta_{2,2k+1} \sin \frac{2\pi kt}{365.25} \right), \\
\xi_t &= \theta_{3,1} + \sum_{k=1}^{K_3} \left(\theta_{3,2k} \cos \frac{2\pi kt}{365.25} + \theta_{3,2k+1} \sin \frac{2\pi kt}{365.25} \right).
\end{aligned}$$

Call this the (K_1, K_2, K_3) model.

Note: This is all for one station. The θ parameters will differ at each station.

Model selection

Use a sequence of likelihood ratio tests

- For each (K_1, K_2, K_3), construct LRT against some (K_1', K_2', K_3'), $K_1' \geq K_1, K_2' \geq K_2, K_3' \geq K_3$ (not all equal) using standard χ^2 distribution theory

- Look at proportion of rejected tests over all stations. If too high, set $K_j = K_j'$ $(j = 1, 2, 3)$ and repeat procedure

- By trial and error, we select $K_1 = 4, K_2 = 2, K_3 = 1$ (17 model parameters for each station)

Models with trend

Add to the above:

- Overall linear trend $\theta_{j,2K+2}t$ added to any of μ_t $(j = 1)$, $\log \psi_t$ $(j = 1)$, ξ_t $(j = 1)$. Define K_j^* to be 1 if this term is included, o.w. 0.

- Interaction terms of form

$$t \cos \frac{2\pi kt}{365.25}, \quad t \sin \frac{2\pi kt}{365.25}, \quad k = 1, ..., K_j^{**}.$$

Typical model denoted

$$(K_1, K_2, K_3) \times (K_1^*, K_2^*, K_3^*) \times (K_1^{**}, K_2^{**}, K_3^{**})$$

Eventually use $(4, 2, 1) \times (1, 1, 0) \times (2, 2, 0)$ model (27 parameters for each station)

Details

- Selection of time-varying threshold — based on the 95th percentile of a 7-day window around the date of interest
- Declustering by r-runs method (Smith and Weissman 1994) — use $r = 1$ for main model runs
- Computation via *sampling the likelihood*: evaluate contributions to likelihood for all observations above threshold, but sample only 5% or 10% of those below, then renormalize to provide accurate approximation to full likelihood

Details (continued)

- Covariances of parameters at different sites:

  $\widehat{\theta}_s$ is MLE at site s, solves $\nabla \ell_s(\widehat{\theta}_s) = 0$

  For two sites s, s',

  $$\mathrm{Cov}\left(\widehat{\theta}_s, \widehat{\theta}_{s'}\right) \approx \left(\nabla^2 \ell_s(\widehat{\theta}_s)\right)^{-1} \mathrm{Cov}\left(\nabla \ell_s(\theta_s), \nabla \ell_{s'}(\theta_{s'})\right) \left(\nabla^2 \ell_{s'}(\widehat{\theta}_{s'})\right)^{-1}$$

  Estimate covariances on RHS empirically, using a subset of days (*same* subset for all stations)

  Also employed when $s = s'$.

  Open question: Should we "regularize" this covariance matrix?

Details (continued)

- Calculating the N-year return value

  For one year ($t = 1, ..., T$), find $y_{\theta,N}$ numerically to solve

$$\sum_{1}^{T} \left(1 + \xi_t \frac{y_{\theta,N} - \mu_t}{\psi_t}\right)^{-1/\xi_t} = -\log\left(1 - \frac{1}{N}\right).$$

  - Also calculate $\frac{\partial y_{\theta,N}}{\partial \theta_j}$ by numerical implementation of inverse function formula
  - Covariances between return level estimates at different sites by

$$\text{Cov}\left\{y_{\widehat{\theta}_s,N}, y_{\widehat{\theta}_{s'},N}\right\} \approx \left(\frac{dy_{\widehat{\theta}_s,N}}{d\theta_s}\right)^T \text{Cov}\left(\widehat{\theta}_s, \widehat{\theta}_{s'}\right) \left(\frac{dy_{\widehat{\theta}_{s'},N}}{d\theta_{s'}}\right).$$

  - Also apply to ratios of return level estimates, such as

$$\frac{25-\text{year return level at } s \text{ in } 1999}{25-\text{year return level at } s \text{ in } 1970}$$

SPATIAL SMOOTHING

Let Z_s be field of interest, indexed by s (typically the logarithm of the 25-year RV at site s, or a log of ratio of RVs. Taking logs improves fit of spatial model, to follow.)

Don't observe Z_s — estimate $\hat{Z}_s$. Assume

$$\begin{aligned} \hat{Z} \mid Z &\sim N[Z, W] \\ Z &\sim N[X\beta, V(\phi)] \\ \hat{Z} &\sim N[X\beta, V(\phi) + W]. \end{aligned}$$

for known W; X are covariates, β are unknown regression parameters and ϕ are parameters of spatial covariance matrix V.

- ϕ by REML
- β given ϕ by GLS
- Predict Z at observed and unobserved sites by kriging

Details

- Covariates
  - Always include intercept
  - Linear and quadratic terms in elevation and log of mean annual precipitation
  - Contrast "climate space" approach of Cooley *et al.* (2007)

Details (continued)

- Spatial covariances
  - Matérn
  - Exponential with nugget
  - Intrinsically stationary model

$$\text{Var}(Z_s - Z_{s'}) \;=\; \phi_1 d_{s,s'}^{\phi_2} + \phi_3$$

  - Matérn with nugget

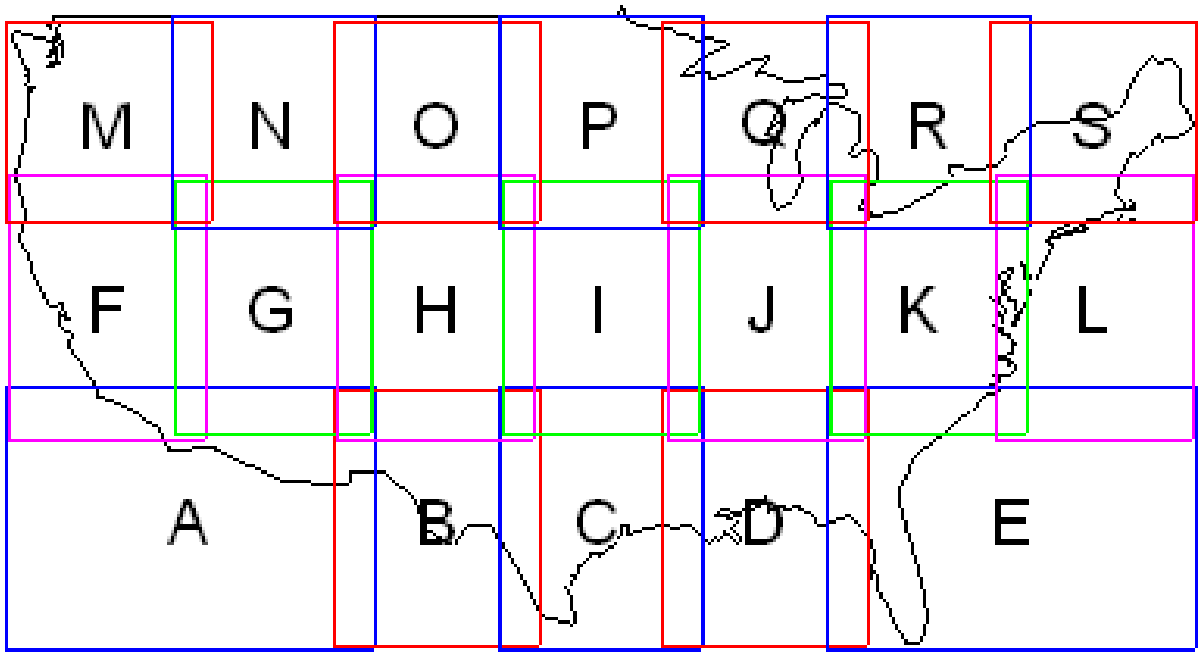
The last-named contains all the previous three as limiting cases and appears to be the best overall, though is often slow to converge (e.g. sometime the range parameter tends to ∞, which is almost equivalent to the intrinsically stationary model)

Details (continued)

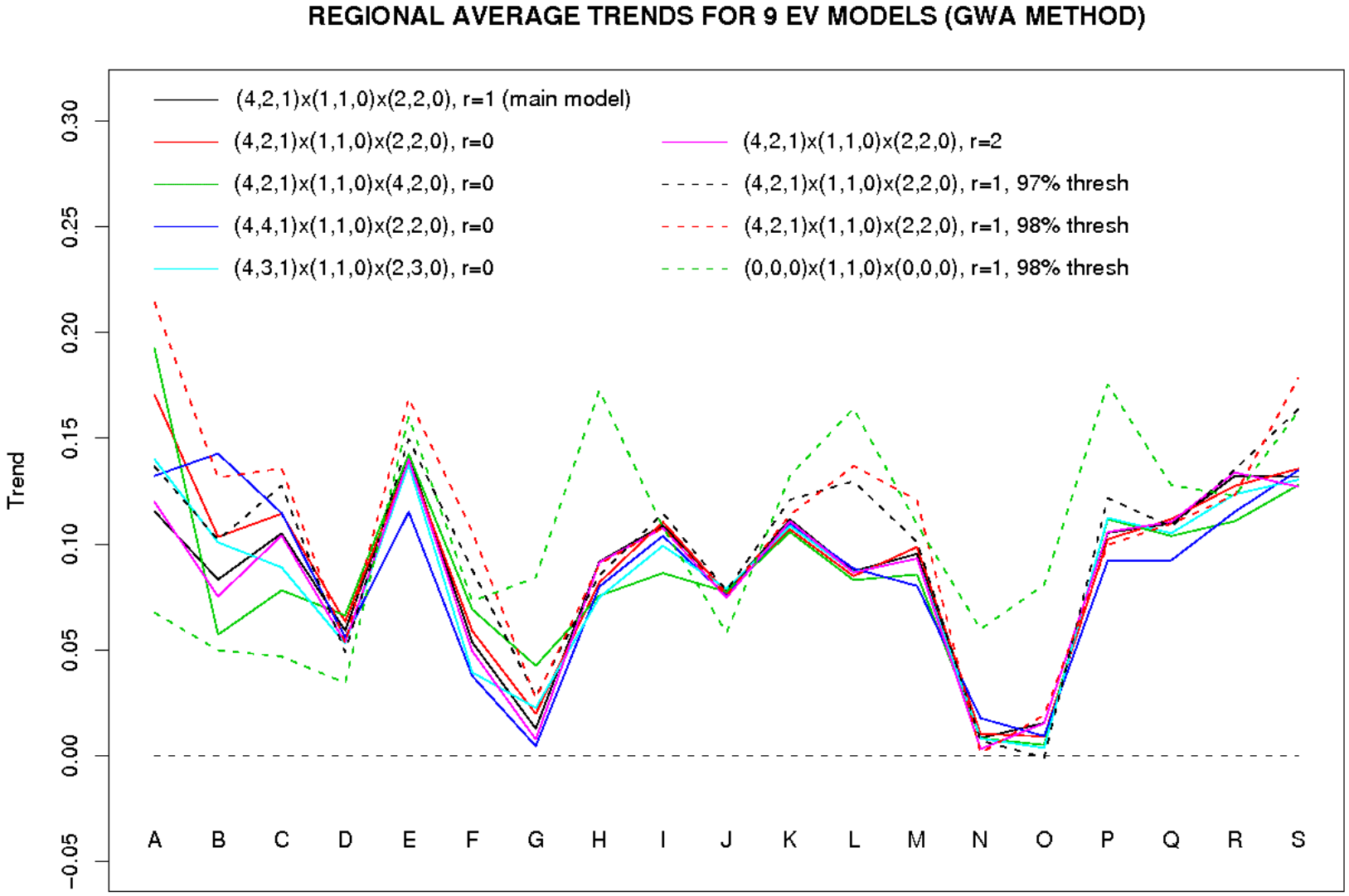
- Spatial heterogeneity

  Divide US into 19 overlapping regions, most $10^o \times 10^o$

  - Kriging within each region
  - Linear smoothing across region boundaries
  - Same for MSPEs
  - Also calculate regional averages, including MSPE



Continental USA divided into 19 regions



Trends across 19 regions (measured as change in log RV25) for 8 different seasonal models and one non-seasonal model with simple linear trends. Regional averaged trends by geometric weighted average approach.

Summary of models shown on previous slide:

1: Preferred covariates model ($r = 0$ for declustering, uses 95% threshold calculated from 7-day window)

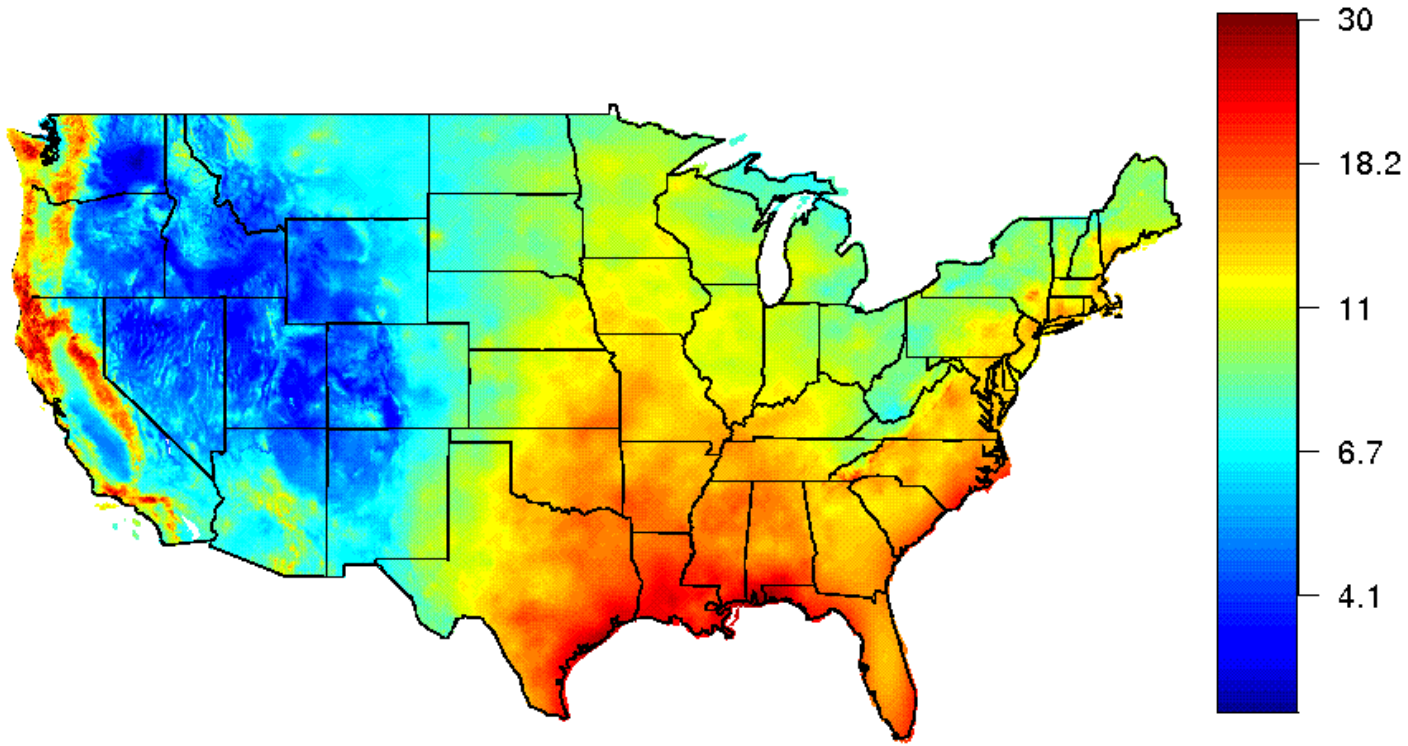
2–4: Three variants where we add covariates to μ_t and/or $\log \psi_t$

5: Replace $r = 0$ by $r = 1$ (subsequent results are based on this model)

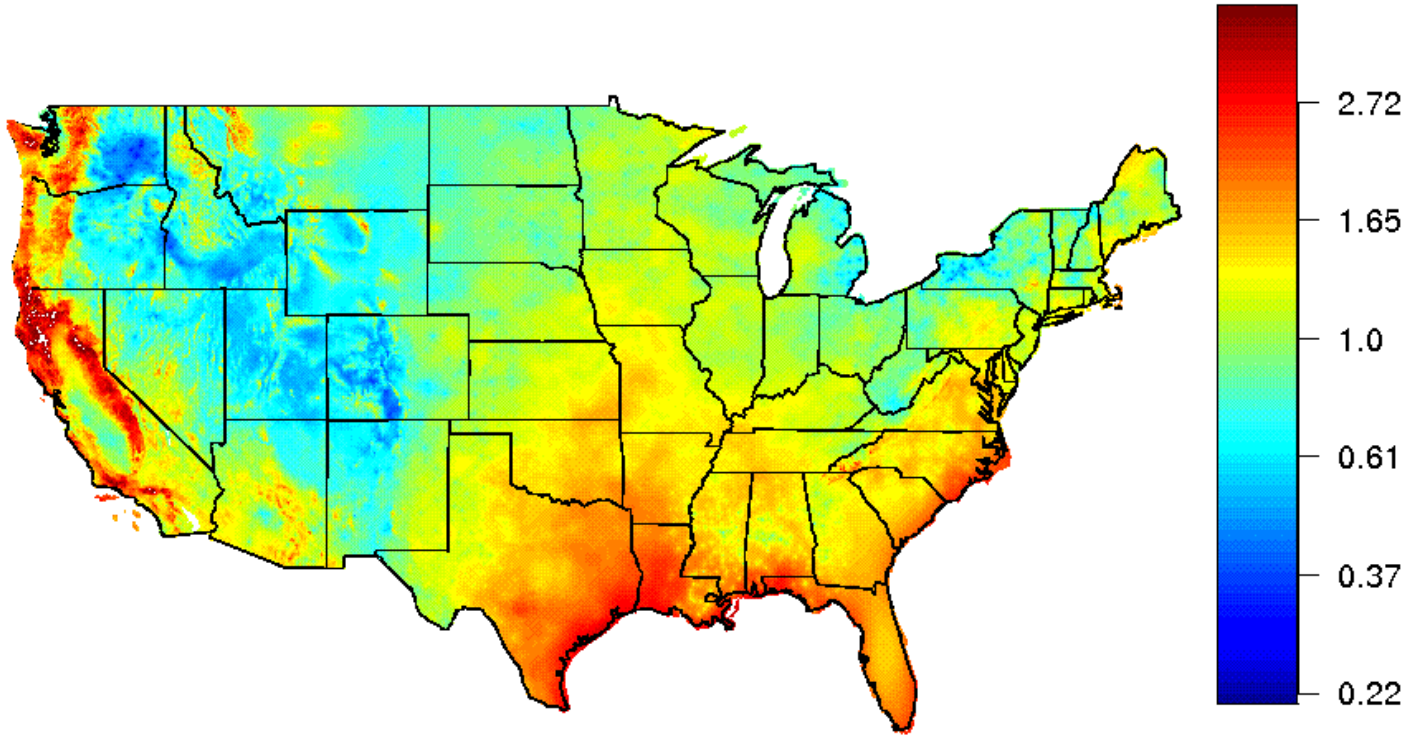
6: Replace $r = 0$ by $r = 2$

7: 97% threshold calculated from 14-day window

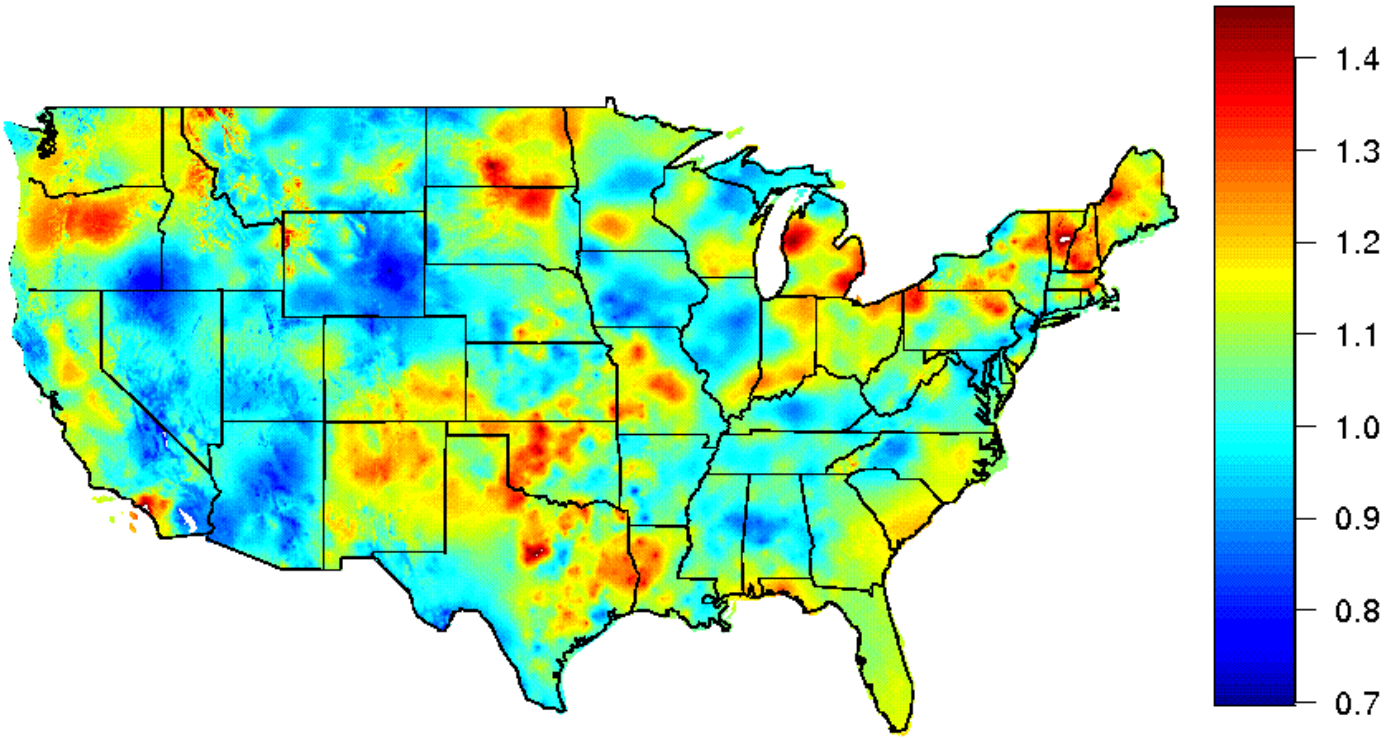
8: 98% threshold calculated from 28-day window



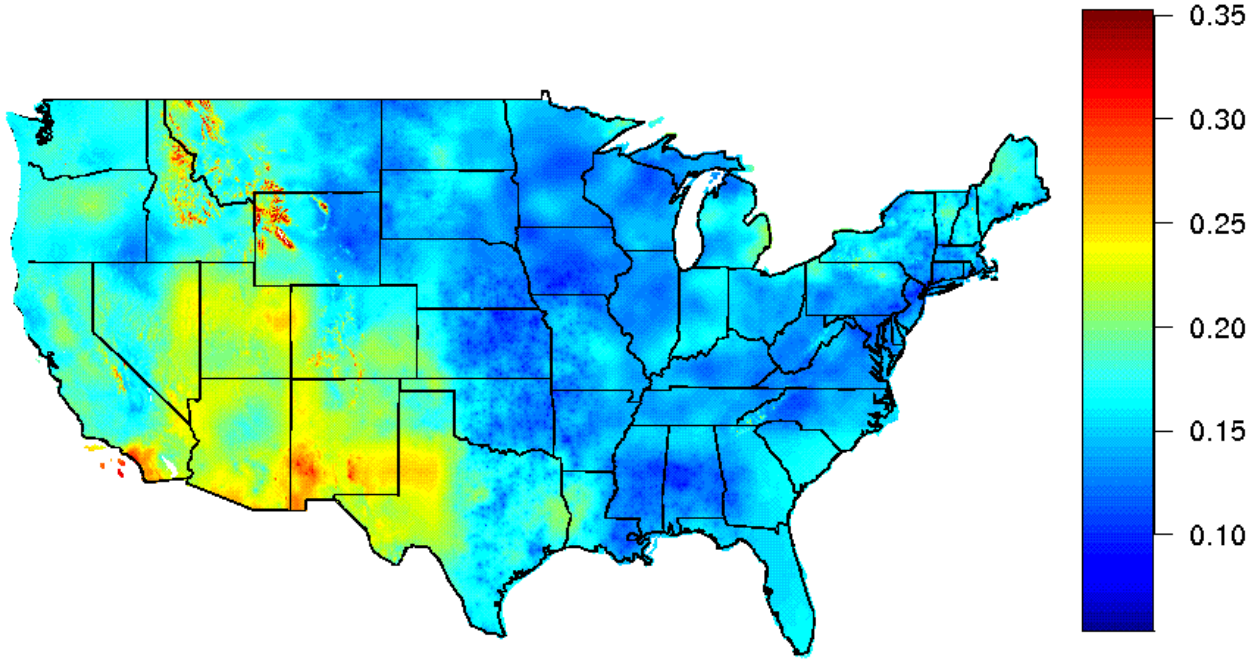
Map of 25-year return values (cm.) for the years 1970–1999



Root mean square prediction errors for map of 25-year return values for 1970–1999



Ratios of return values in 1999 to those in 1970



Root mean square prediction errors for map of ratios of 25-year return values in 1999 to those in 1970

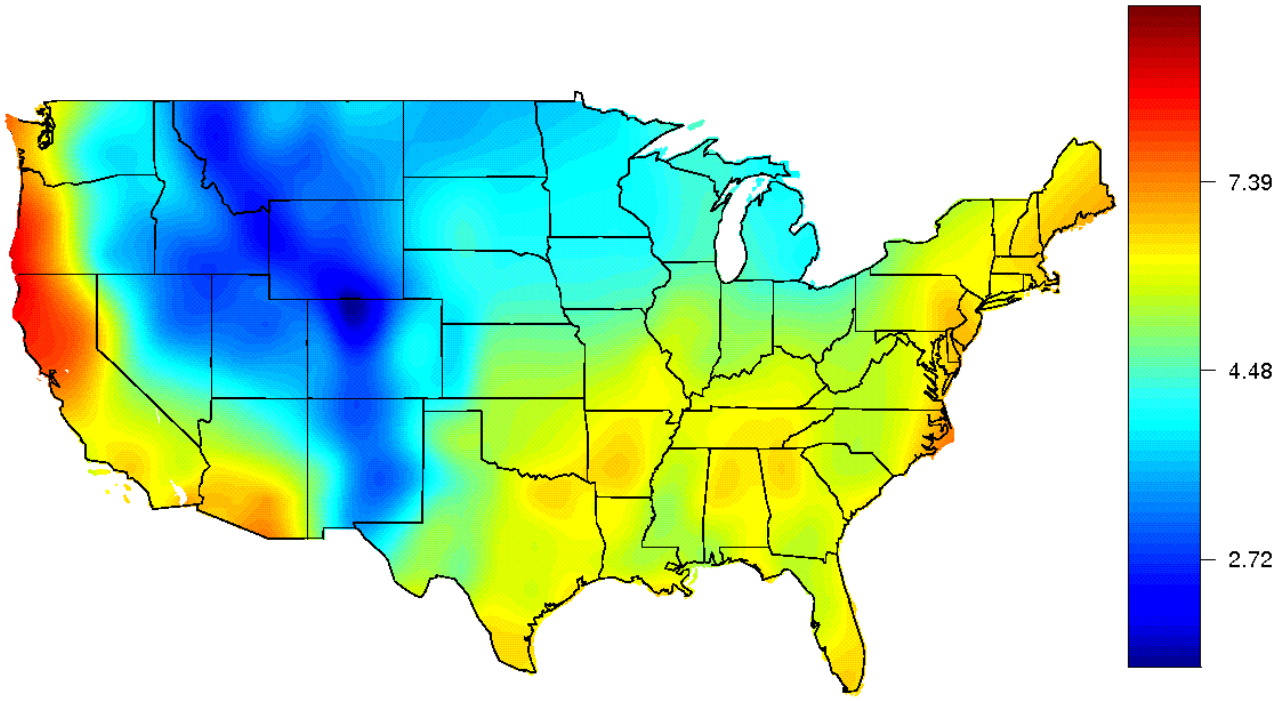
	Δ_1	S_1	Δ_2	S_2		Δ_1	S_1	Δ_2	S_2
A	−0.01	.03	0.05**	.05	K	0.08***	.01	0.09**	.03
B	0.07**	.03	0.08***	.04	L	0.07***	.02	0.07*	.04
C	0.11***	.01	0.10	.03	M	0.07***	.02	0.10**	.03
D	0.05***	.01	0.06	.05	N	0.02	.03	0.01	.03
E	0.13***	.02	0.14*	.05	O	0.01	.02	0.02	.03
F	0.00	.02	0.05*	.04	P	0.07***	.01	0.11***	.03
G	−0.01	.02	0.01	.03	Q	0.07***	.01	0.11***	.03
H	0.08***	.01	0.10***	.03	R	0.15***	.02	0.13***	.03
I	0.07***	.01	0.12***	.03	S	0.14***	.02	0.12*	.06
J	0.05***	.01	0.08**	.03					

Δ_1: Mean change in log 25-year return value (1970 to 1999) by kriging

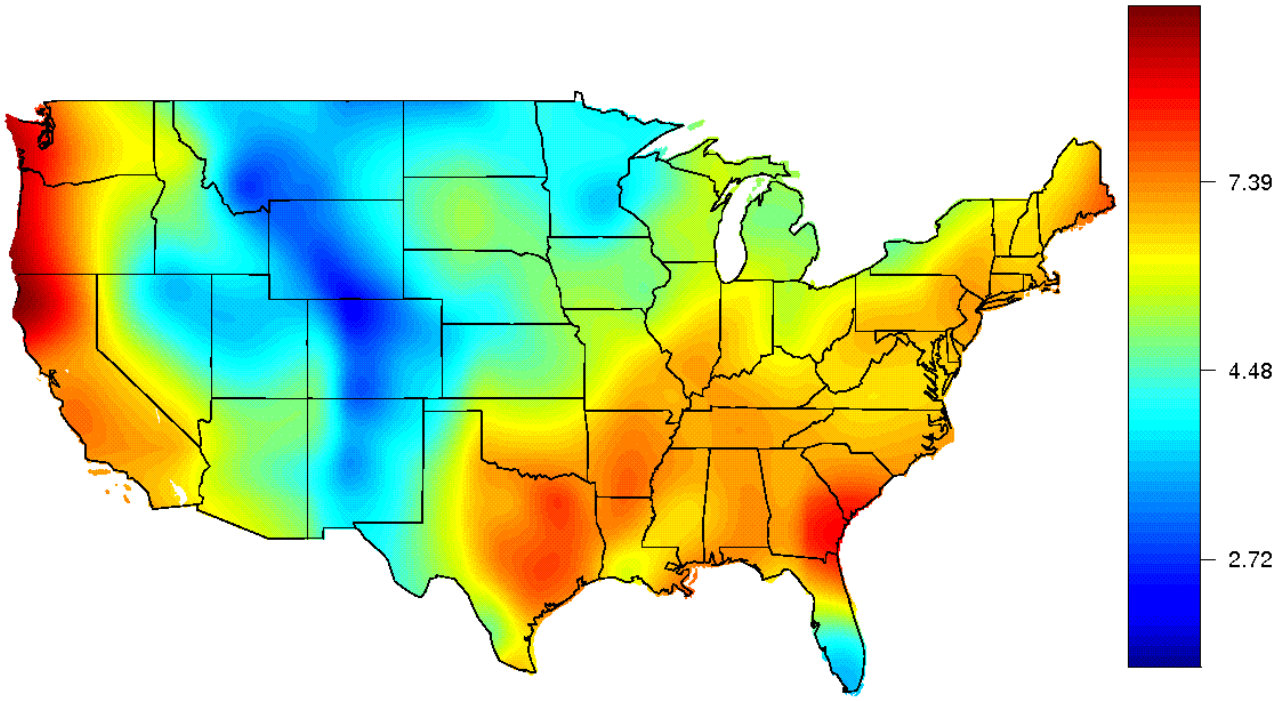
S_1: Corresponding standard error (or RMSPE)

Δ_2, S_2: same but using geometrically weighted average (GWA)

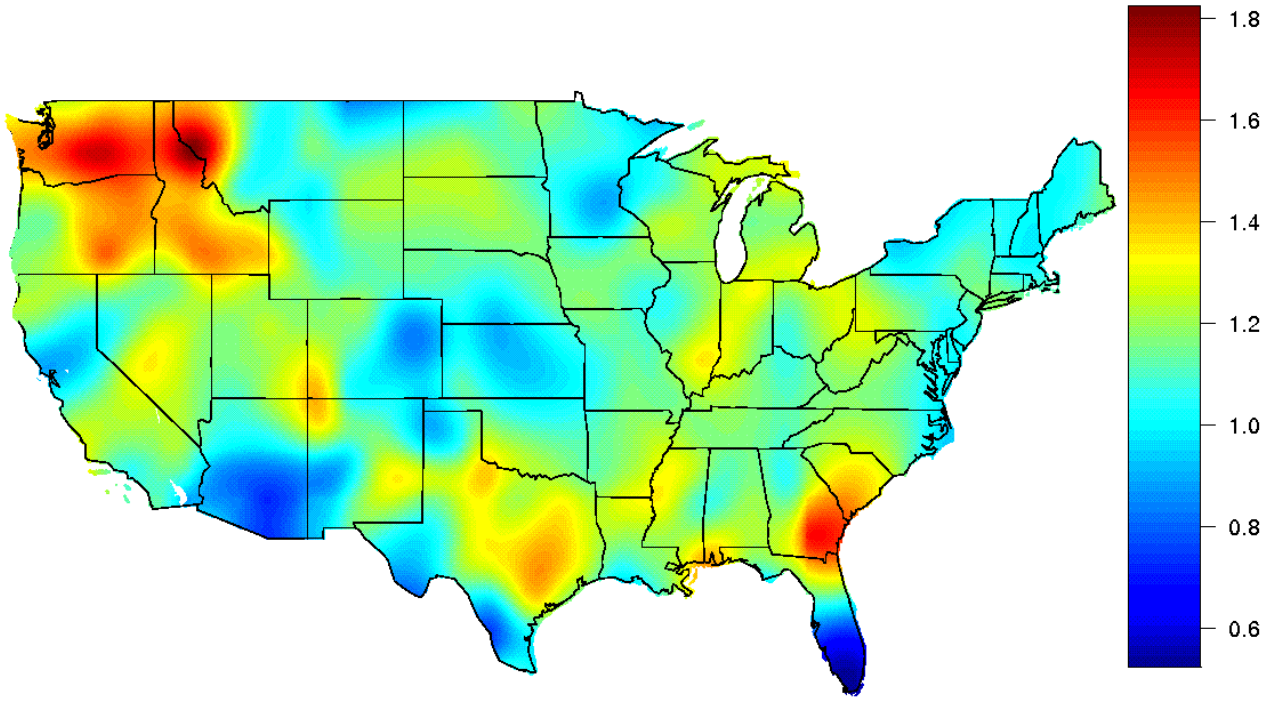
Stars indicate significance at 5%*, 1%**, 0.1%***.



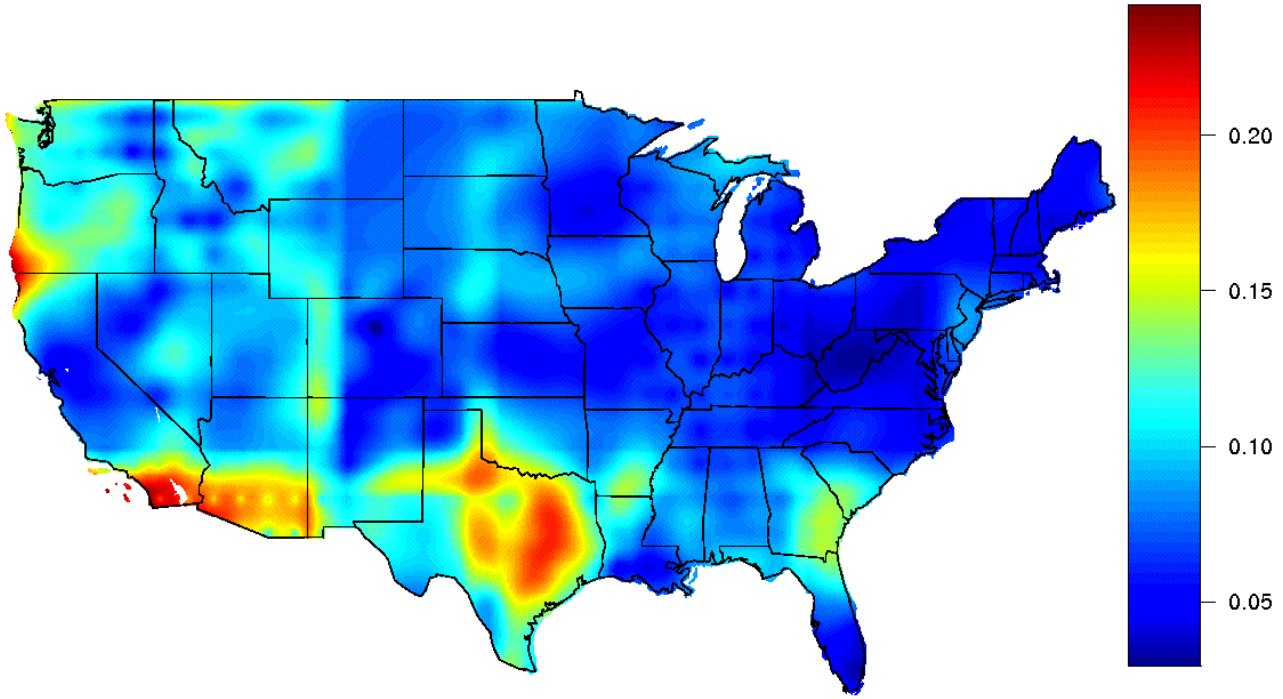
Return value map for CCSM data (cm.): 1970–1999



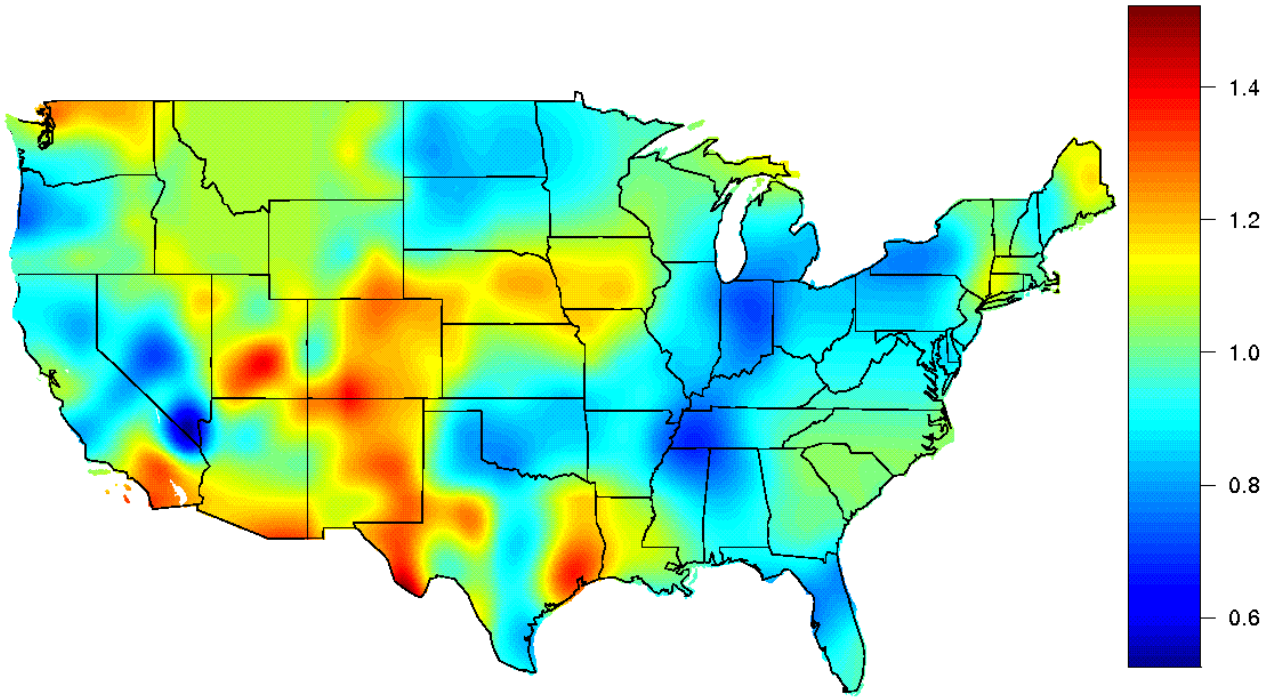
Return value map for CCSM data (cm.): 2070–2099



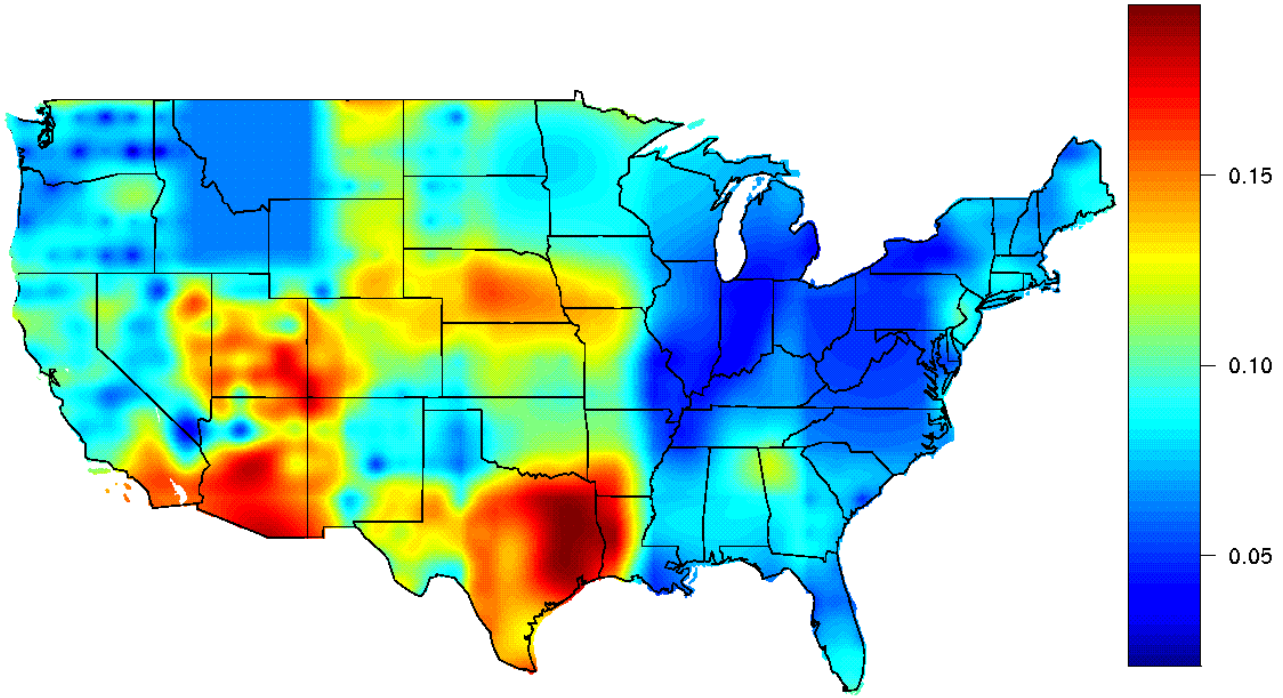
Estimated ratios of 25-year return values for 2070–2099 to those of 1970–1999, based on CCSM data, A2 scenario



RMSPE for map in previous slide



Extreme value model with trend: ratio of 25-year return value in 1999 to 25-year return value in 1970, based on CCSM data



RMSPE for map in previous slide

	Δ_3	S_3	Δ_4	S_4		Δ_3	S_3	Δ_4	S_4
A	0.16^{**}	.07	0.24^{**}	.10	K	-0.08^{***}	.02	-0.11^{*}	.05
B	0.14^{***}	.04	0.12^{***}	.06	L	-0.04	.04	-0.03	.06
C	0.02	.05	-0.14	.11	M	0.01	.03	0.00	.08
D	-0.06	.04	-0.15	.10	N	0.06^{**}	.02	0.05	.06
E	-0.07^{*}	.03	-0.09	.08	O	-0.03	.04	-0.06	.07
F	-0.07^{*}	.04	-0.03	.05	P	-0.01	.04	-0.07	.07
G	0.03	.03	0.08^{*}	.04	Q	-0.04	.04	-0.03	.07
H	0.11^{***}	.03	0.08	.06	R	-0.17^{***}	.03	-0.06	.08
I	-0.02	.04	-0.05	.07	S	0.00	.04	0.02	.05
J	-0.15^{***}	.03	-0.16^{**}	.06					

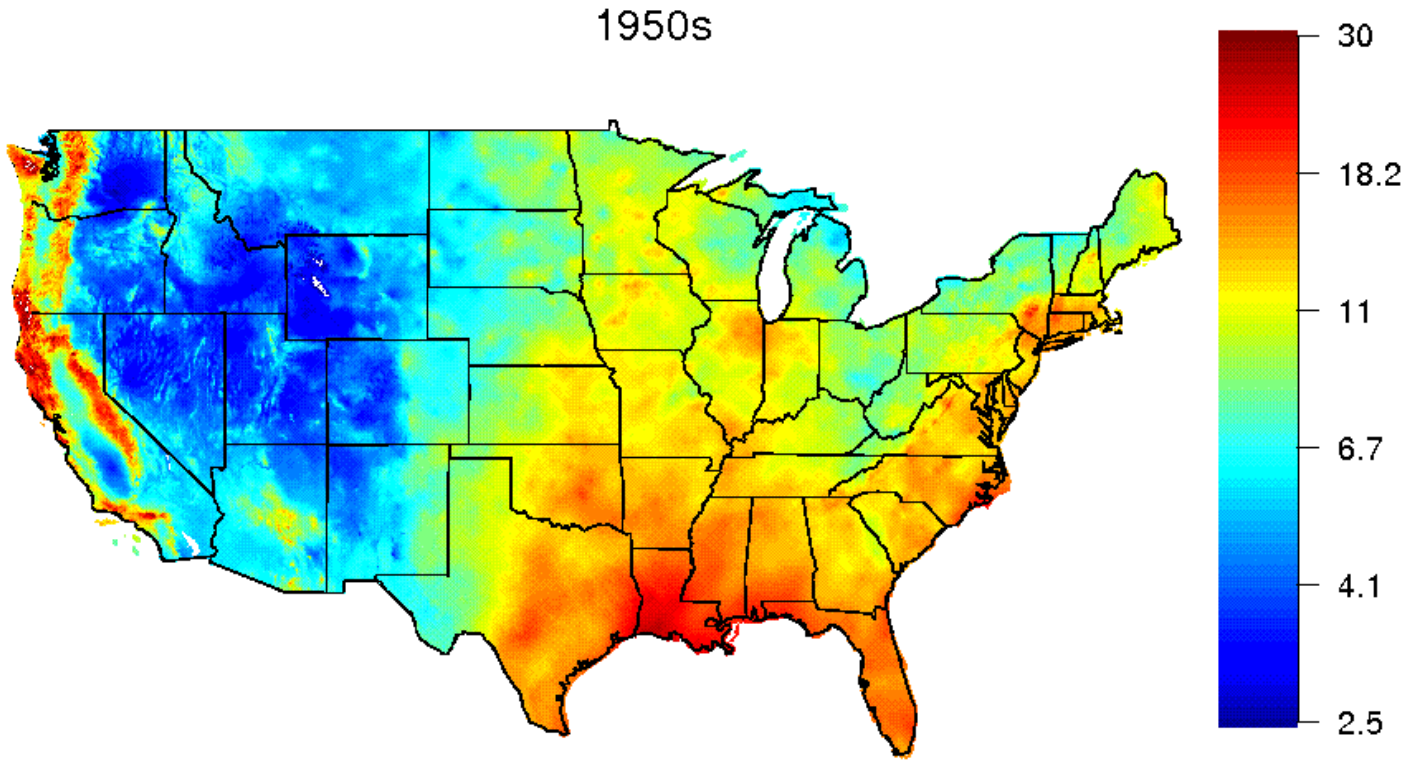
Δ_3: Mean change in log 25-year return value (1970 to 1999) for CCSM, by kriging

SE_3: Corresponding standard error (or RMSPE)

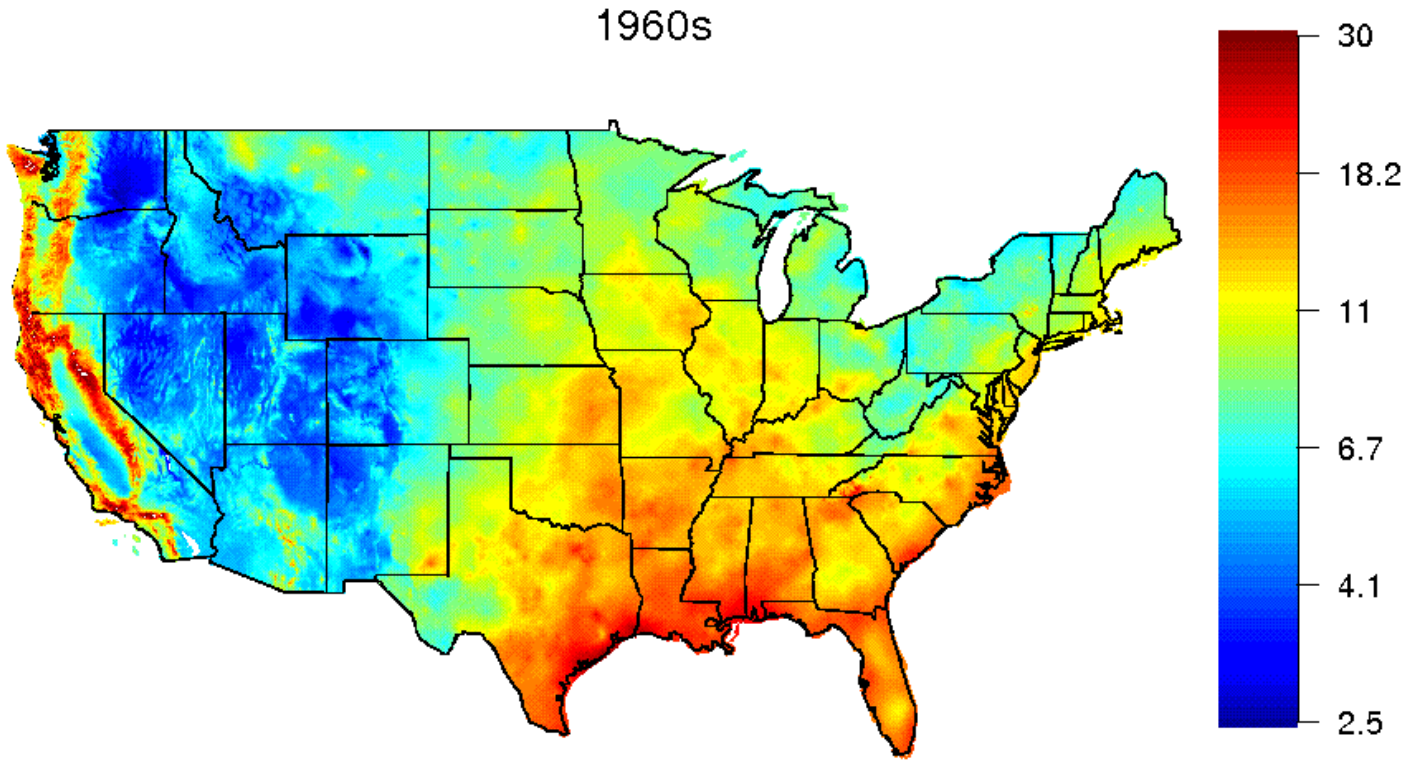
Δ_4, SE_4: Results using GWA

Stars indicate significance at 5%*, 1%**, 0.1%***.

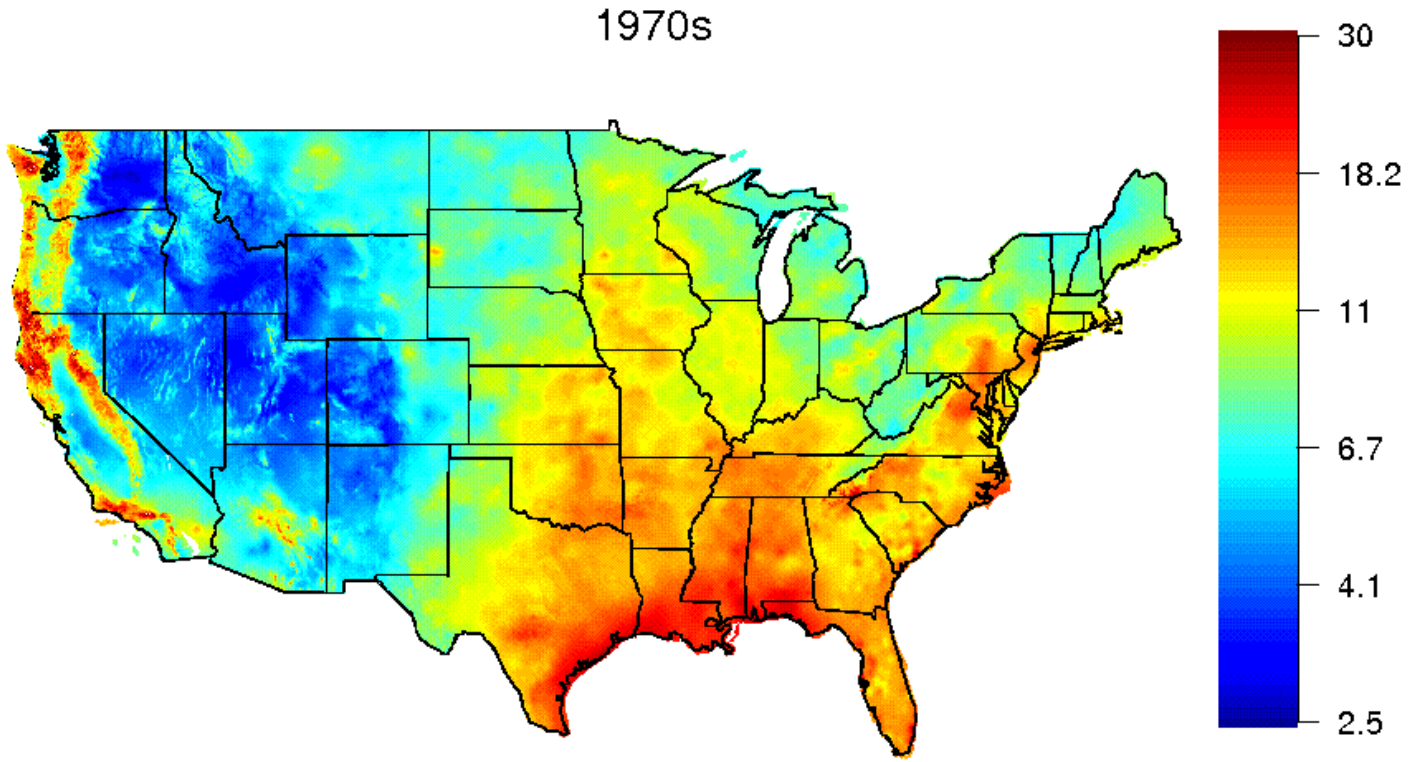
RETURN VALUE MAPS FOR INDIVIDUAL DECADES



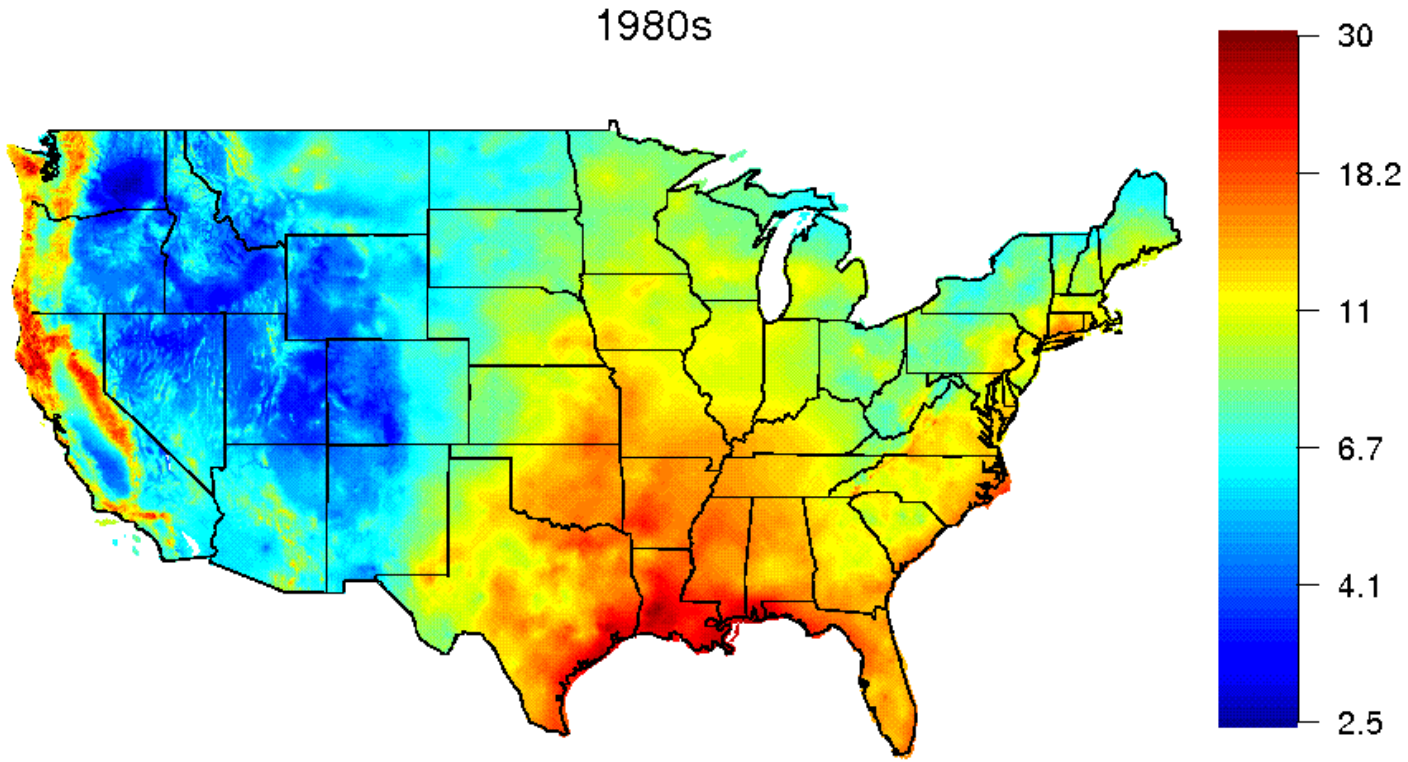
Map of 25-year return values (cm.) for the years 1950–1959



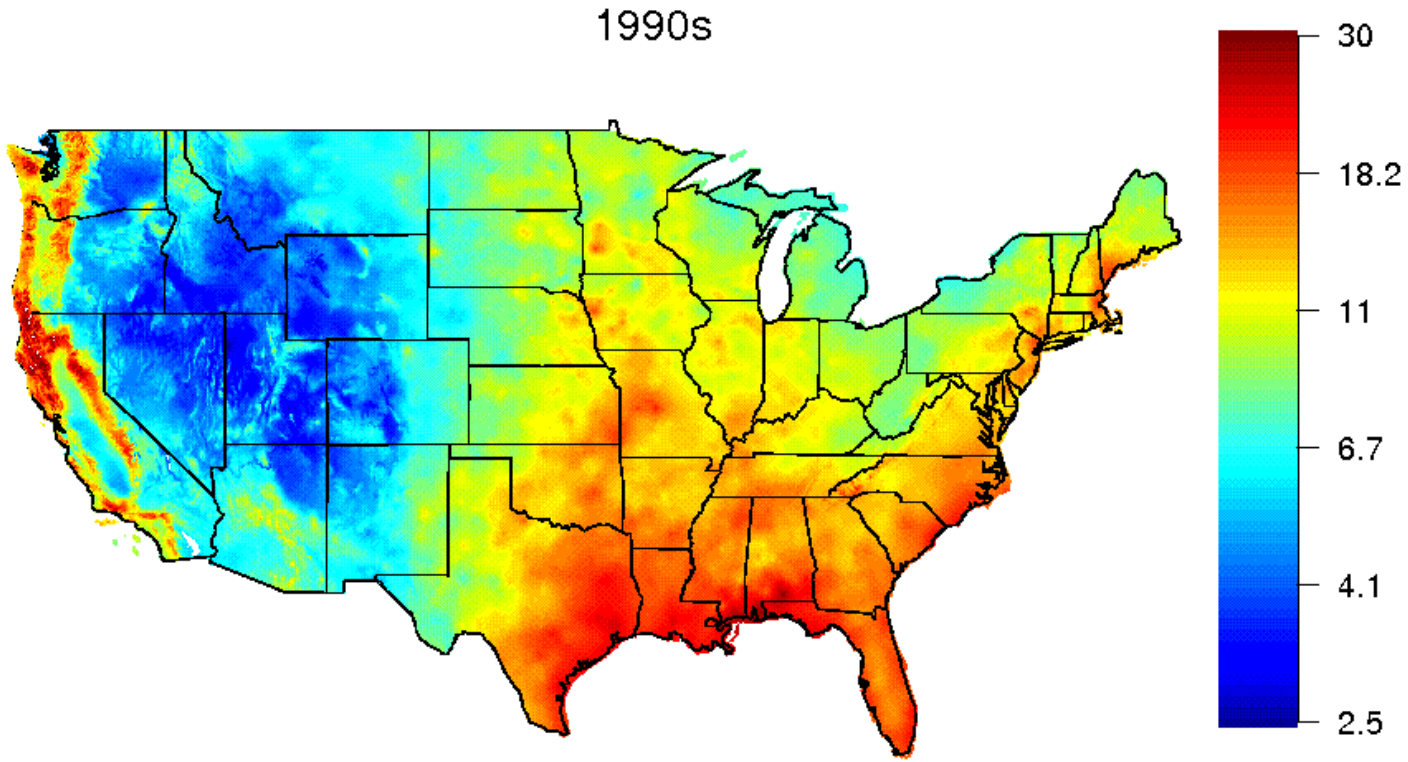
Map of 25-year return values (cm.) for the years 1960–1969



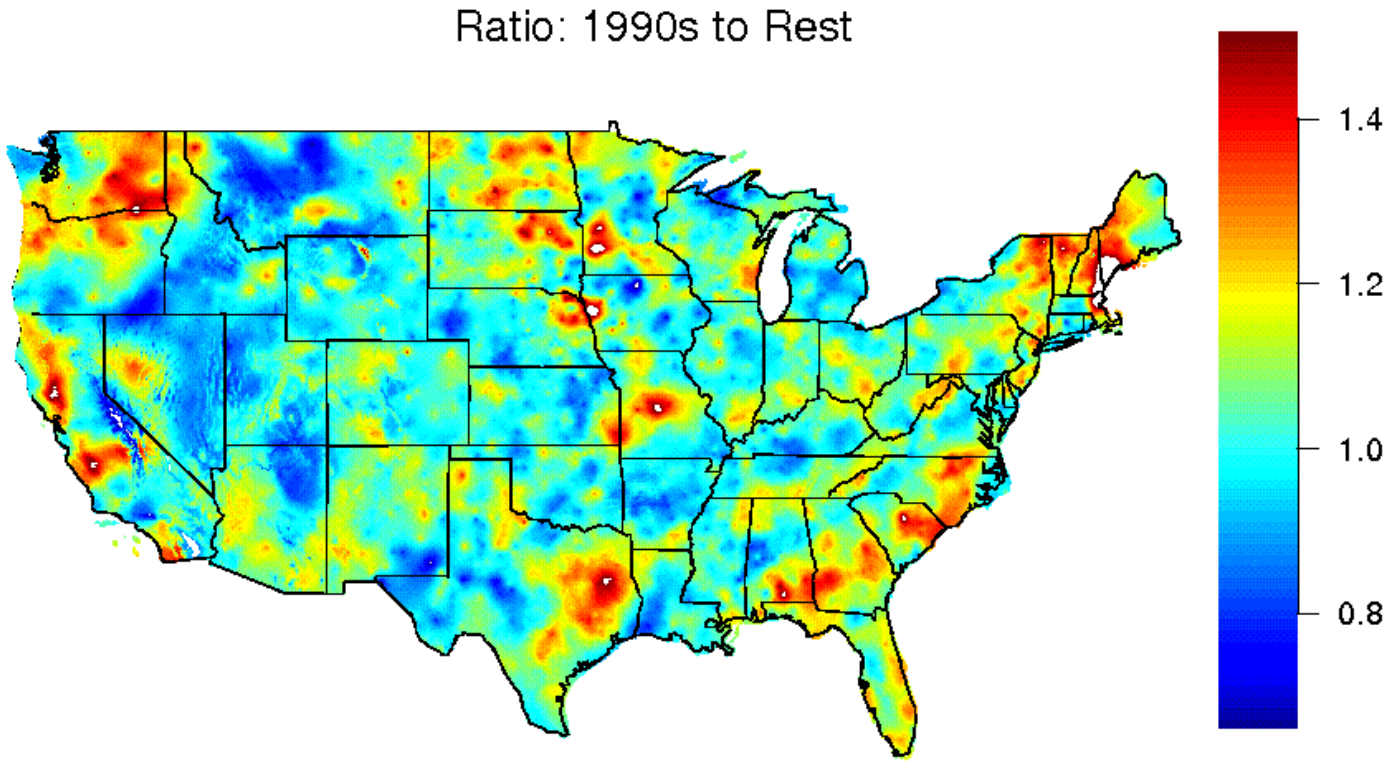
Map of 25-year return values (cm.) for the years 1970–1979



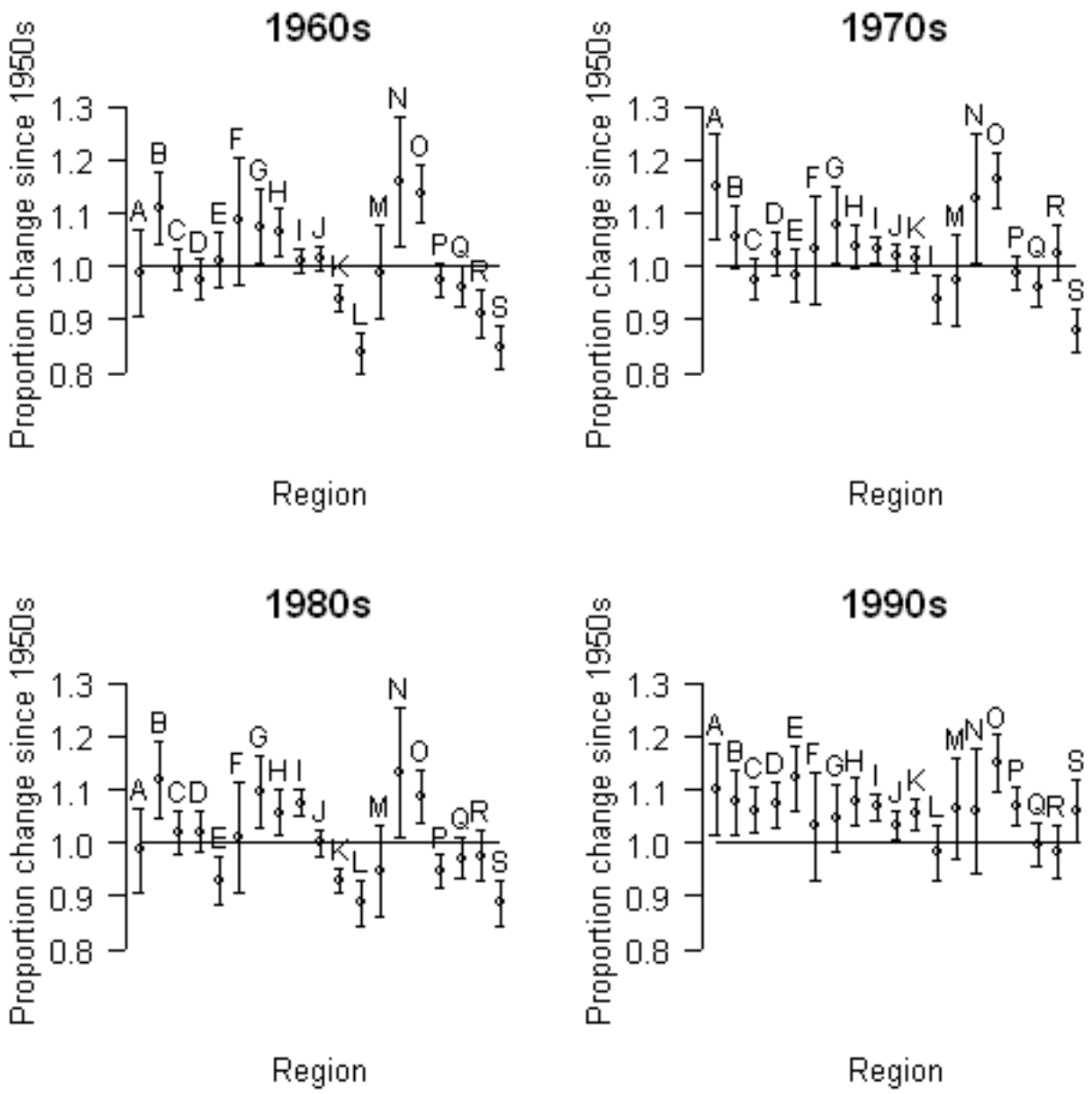
Map of 25-year return values (cm.) for the years 1980–1989



Map of 25-year return values (cm.) for the years 1990–1999



Estimated ratios of 25-year return values for 1990s compared with average at each location over 1950–1989



Regional changes in log RV25 for each decade compared with 1950s

CONCLUSIONS

1. Focus on N-year return values — strong historical tradition for this measure of extremes (we took $N = 25$ here)
2. Seasonal variation of extreme value parameters is a critical feature of this analysis
3. Overall significant increase over 1970–1999 except for parts of western states — average increase across continental US is 7%
4. Kriging better than GWA
5. *But...* based on CCSM data there is a completely different spatial pattern and no overall increase
6. Projections to 2070–2099 show further strong increases but note caveat based on point 5
7. Decadal variations since 1950s show strongest increases during 1990s.