DETECTION AND ATTRIBUTION IN CLIMATE SCIENCE

Richard L Smith

Departments of STOR and Biostatistics, University of North Carolina at Chapel Hill

and

Statistical and Applied Mathematical Sciences Institute (SAMSI)

SAMSI Graduate Class

September 5, 2017

THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

samsi
NSF Duke NCSU UNC

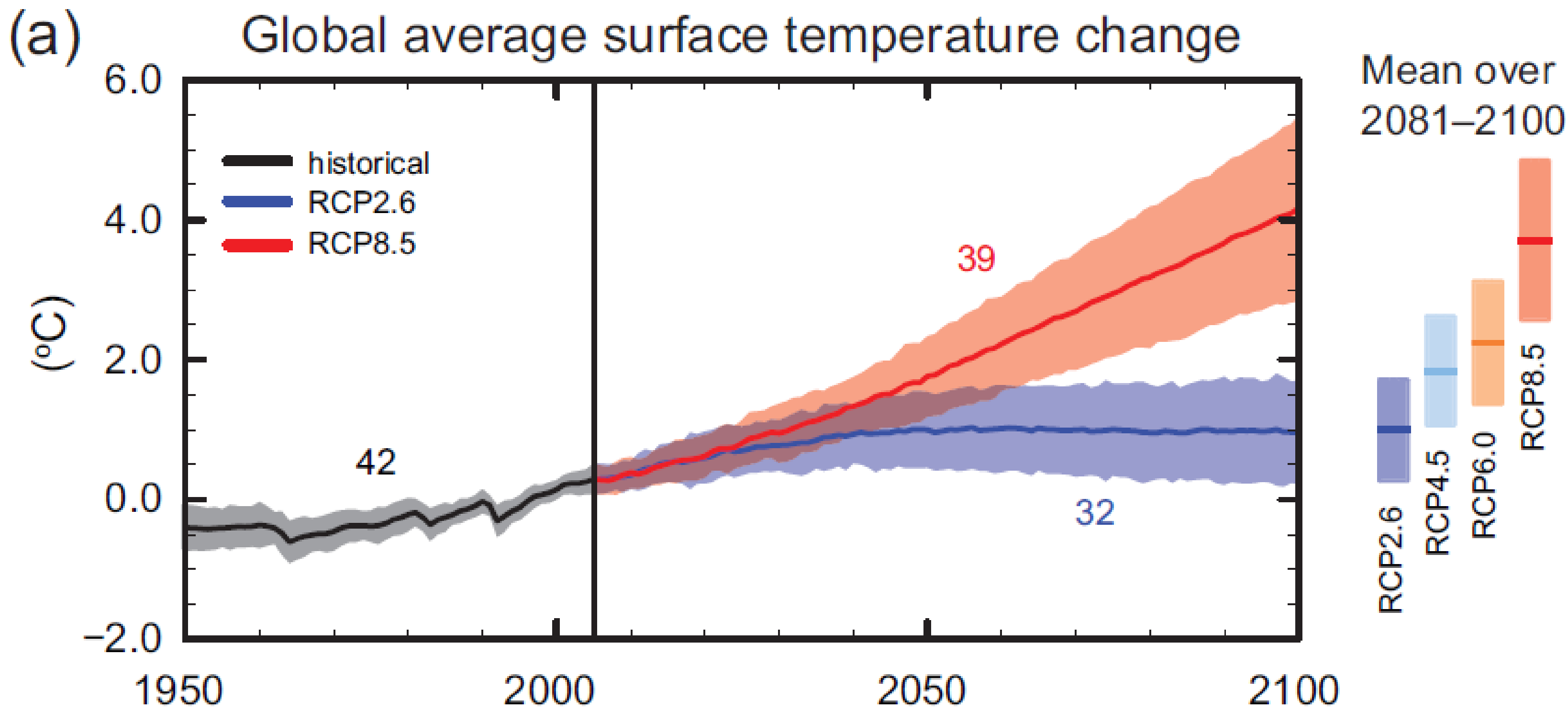


Figure SPM.7 | CMIP5 multi-model simulated time series from 1950 to 2100 for (a) change in global annual mean surface temperature relative to 1986–2005, (b) Northern Hemisphere September sea ice extent (5-year running mean), and (c) global mean ocean surface pH. Time series of projections and a measure of uncertainty (shading) are shown for scenarios RCP2.6 (blue) and RCP8.5 (red). Black (grey shading) is the modelled historical evolution using historical reconstructed forcings. The mean and associated uncertainties averaged over 2081–2100 are given for all RCP scenarios as colored vertical bars. The numbers of CMIP5 models used to calculate the multi-model mean is indicated.

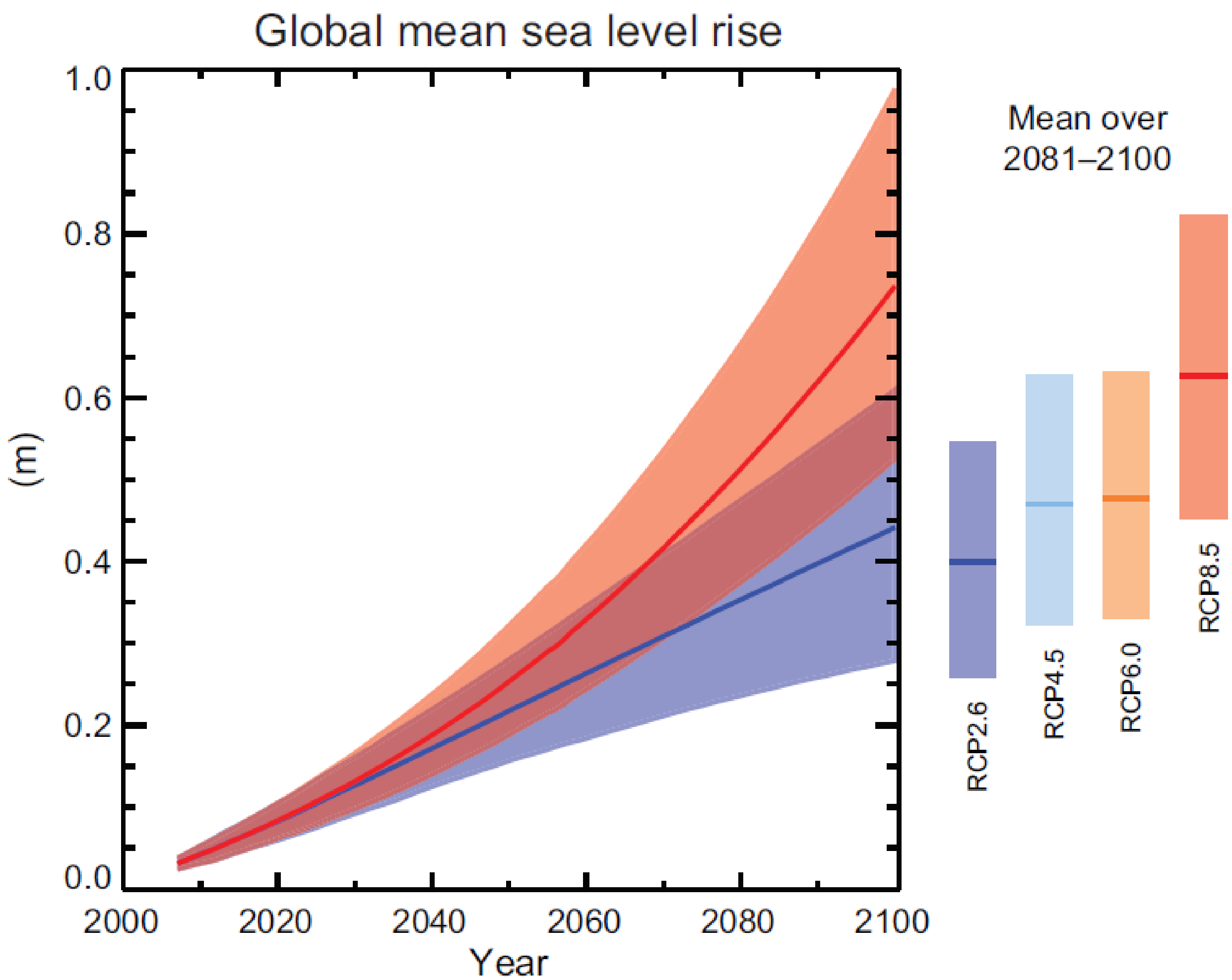


Figure SPM.9 | Projections of global mean sea level rise over the 21st century relative to 1986–2005 from the combination of the CMIP5 ensemble with process-based models, for RCP2.6 and RCP8.5. The assessed *likely* range is shown as a shaded band. The assessed *likely* ranges for the mean over the period 2081–2100 for all RCP scenarios are given as coloured vertical bars, with the corresponding median value given as a horizontal line. For further technical details see the Technical Summary Supplementary Material {Table 13.5, Figures 13.10 and 13.11; Figures TS.21 and TS.22}

From the 2014 IPCC Report (Summary for Policymakers):

"It is *extremely likely** that more than half of the observed increase in global average surface temperature from 1951 to 2010 was caused by anthropogenic increase in greenhouse gas concentrations and other anthropogenic forcings together."

What does this mean? How does IPCC evaluate statements of this nature?

*probability greater than 95%

Introduction to Detection and Attribution (1)

Detection and Attribution refers to a class of statistical techniques that are used to break down a *climate signal* (temperatures, precipitation, wind speeds, etc.) into a series of components due to various *forcing factors*.

Typical forcing factors that are considered include greenhouse gases, other anthropogenic components (including aerosols, which tend to have a cooling effect), variations in solar output and volcanic eruptions. The last two are considered *natural forcings*.

A forcing factor is said to be *detected* if there is a statistically significant contribution based on that factor in the observational signal.

Among the factors that are detected, the *attributions* of those factors are numerical coefficients that represent the contributions of the individual factors to the overall signal.

Introduction to Detection and Attribution (2)

Detection and Attribution is largely a statistical technique developed by atmospheric scientists, but during the present decade has attracted more attention from statisticians.

Its origins are usually attributed to a paper by Hasselmann (1979) but the concept was reformulated and greatly extended during the 1990s and 2000s.

During the present decade, there have been a number of attempts to extend the statistical foundations of the method.

This presentation reviews some of the history and background of this methodology, leading up to a recent paper by Katzfuss, Hammerling and Smith (*Geophysical Research Letters*, 2017).

Hasselmann's First Approach (1979)

On the signal-to-noise problem in atmospheric response studies

By KLAUS HASSELMANN
Max-Planck-Institut für Meteorologie, Hamburg

Summary

The problem of identifying the mean atmospheric response to external forcing in the presence of the natural variability of the atmosphere is treated as a pattern-detection problem. It is shown that without application of filtering techniques to reduce the number of degrees of freedom of the response pattern the atmospheric response inferred from data or model experiments will normally fail a multi-variate significance test. A step-wise pattern construction method is proposed which avoids these difficulties. Starting from a given set of anticipated response patterns, a transformed set of patterns is derived which, used as a truncated basis set to represent the observed response, maximizes the statistical significance of the response. The patterns are ordered *a priori* in a sequence reflecting their anticipated contribution to the total response, the sequence being terminated when the net response falls below a prescribed significance level. In effect the method filters out the statistically significant components of the atmospheric response. For application to model experiments a multi-variate analysis of the low-frequency model variability is required.

- Overall change (e.g. in temperature field) represented by n-dimensional vector $\bar{\boldsymbol{\Phi}}$.
- Estimated change from data: $\boldsymbol{\Phi}$
- Assume $\boldsymbol{\Phi} - \bar{\boldsymbol{\Phi}} \sim \mathcal{N}(0, \mathbf{C})$
- $\mathbf{C}$ estimated from data but *treated as known*
- Test $H_0: \; \bar{\boldsymbol{\Phi}} = 0 \implies \chi^2$ test

Hasselmann's First Approach (continued)

- Suppose $\bar{\mathbf{\Phi}} = B\bar{\mathbf{\Psi}}$ where B is a $n \times p$ matrix of known basis functions (interpreted as a p-dimensional "signal")
- A revised estimate $\tilde{\mathbf{\Phi}}$ is chosen to minimize $|\tilde{\mathbf{\Phi}} - \mathbf{\Phi}|^2$. This in turn is used to construct a revised χ^2 text statistic.
- A key part of the method is *expansion in principal components (EOFs)*. Hasselmann anticipated that it might in practice be necessary to restrict to a small number of leading EOFs (he suggested between 5 and 20).

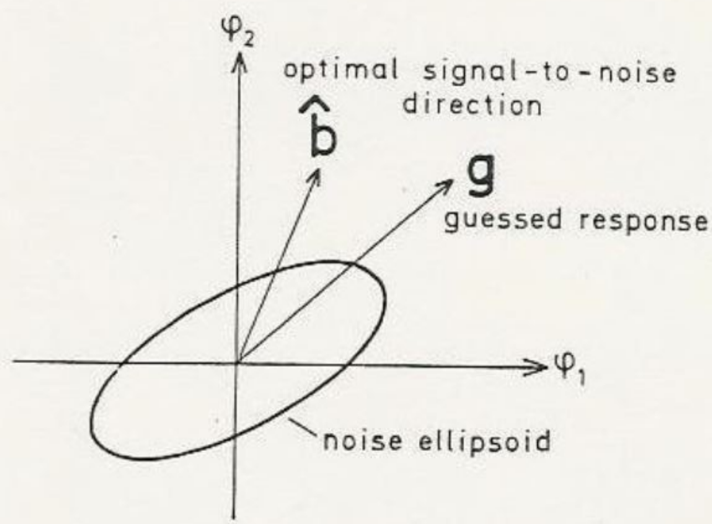


Figure 1. Relation between guessed response pattern vector g and maximal significance direction $\hat{\mathfrak{b}}$ relative to error ellipsoid, see Eq. (27).

Extensions

- The initial paper of Hasselmann was followed by a number of extensions and ramafications in the 1990s, e.g. Hasselmann (1993), *Journal of Climate*, Hasselmann (1997), *Climate Dynamics*, Hegerl and North (1997), *Journal of Climate*, North and Stevens (1998), *Journal of Climate*.

- It was designed to be highly multidimensional

- However it was also implicit that a reduction in dimension (via leading EOFs) was needed to make the method applicable in practice

The method comes to maturity: Two papers in 1996

A search for human influences on the thermal structure of the atmosphere

B. D. Santer*, K. E. Taylor*†, T. M. L. Wigley‡, T. C. Johns§, P. D. Jones‖, D. J. Karoly¶, J. F. B. Mitchell§, A. H. Oort#, J. E. Penner†, V. Ramaswamy#, M. D. Schwarzkopf#, R. J. Stouffer# & S. Tett§

* Program for Climate Model Diagnosis and Intercomparison, † Atmospheric Science Division, Lawrence Livermore National Laboratory, Livermore, California 94550, USA
‡ National Center for Atmospheric Research, Boulder, Colorado 80307-3000, USA
§ Hadley Centre for Climate Prediction and Research, Meteorological Office, Bracknell RG12 2SY, UK
‖ Climatic Research Unit, University of East Anglia, Norwich NR4 7TJ, UK
¶ Cooperative Research Centre for Southern Hemisphere Meteorology, Monash University, Clayton VIC 3168, Australia
# NOAA/Geophysical Fluid Dynamics Laboratory, PO Box 308, Princeton University, Princeton, New Jersey 08542, USA

The observed spatial patterns of temperature change in the free atmosphere from 1963 to 1987 are similar to those predicted by state-of-the-art climate models incorporating various combinations of changes in carbon dioxide, anthropogenic sulphate aerosol and stratospheric ozone concentrations. The degree of pattern similarity between models and observations increases through this period. It is likely that this trend is partially due to human activities, although many uncertainties remain, particularly relating to estimates of natural variability.

NATURE · VOL 382 · 4 JULY 1996

OCTOBER 1996 HEGERL ET AL. 2281

Journal of Climate

Detecting Greenhouse-Gas-Induced Climate Change with an Optimal Fingerprint Method

GABRIELE C. HEGERL,* HANS VON STORCH,* KLAUS HASSELMANN,* BENJAMIN D. SANTER,+ ULRICH CUBASCH,& AND PHILIP D. JONES@

*Max-Planck-Institut für Meteorologie, Hamburg, Germany.
+PCMDI/Lawrence Livermore National Laboratory, Livermore, California.
&Deutsches Klimarechenzentrum, Hamburg, Germany.
@Climatic Research Unit, University of East Anglia, Norwich, United Kingdom.

(Manuscript received 26 August 1994, in final form 20 March 1996)

Santer et al. (1996)

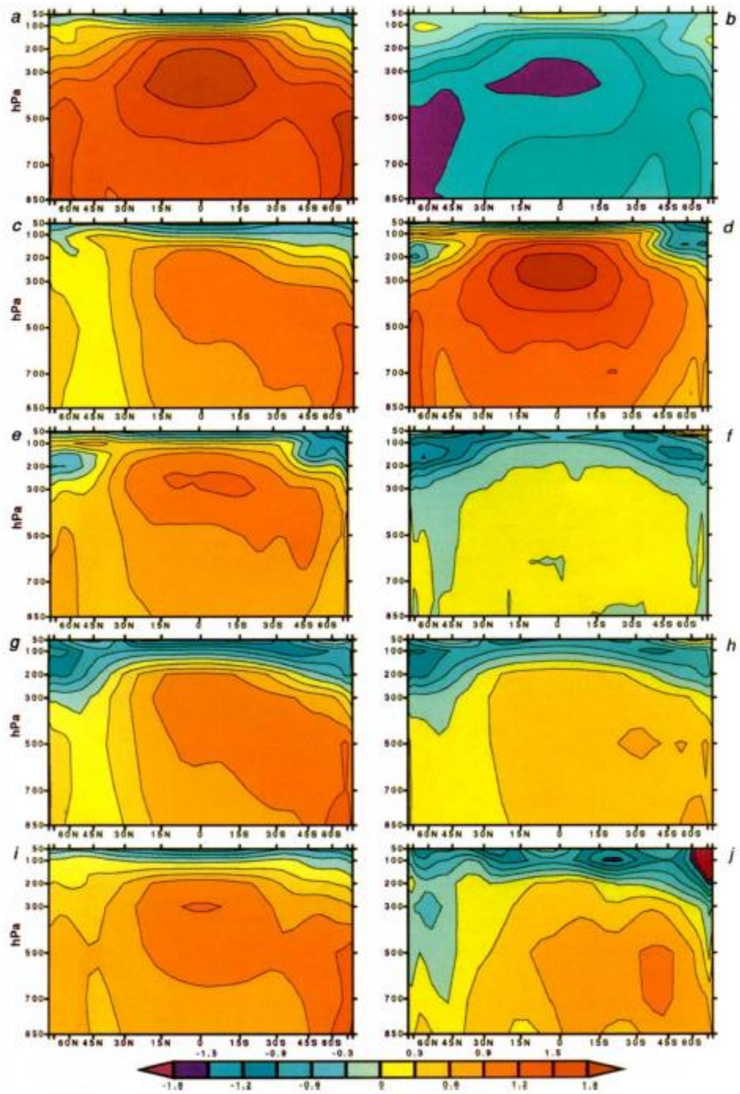


FIG. 1 Modelled and observed zonal-mean annually averaged changes (°C) in the thermal structure of the atmosphere. The equilibrium experiments by Taylor and Penner (TP)[9] simulate temperature changes for nominal 'present-day' levels of atmospheric CO_2 only (C-TP; *a*), anthropogenic sulphate aerosols only (S-TP; *b*), and combined forcing by CO_2 + sulphate aerosols (SC-TP; *c*) relative to a control run with pre-industrial levels of CO_2 and no anthropogenic sulphur emissions. All TP integrations were at least 30 years in duration, and temperature-change signals were computed using averages over the last 20 years of the control run and each perturbation experiment. Patterns of the response to time-varying increases in greenhouse gases only (C-HC; *d*) and in greenhouse gases and aerosols (SC-HC; *e*) were taken from simulations performed with the Hadley Centre CGCM[10,20]. Temperature-change signals are the decadal averages of C-HC and SC-HC for the modelled '1990s' expressed relative to the respective C-HC and SC-HC averages over 1880–1920. The possible effects of stratospheric ozone reduction over the period 1979–90 (*f*) are from a recent equilibrium experiment by Ramaswamy *et al.*[15] The sensitivity studies COMB1 (SC-TP + O_3; *g*) and COMB2 ($\frac{1}{2}$SC-TP + O_3; *h*) consider the possible effects of stratospheric O_3 depletion on the SC-TP signal. COMB3 ($\frac{1}{2}$S-TP + C-TP; *i*) illustrates the sensitivity of model-observed pattern similarities to a possible overestimate of direct aerosol effects in TP. Observed changes (*j*) are radiosonde-based temperature measurements from the data set by Oort[8], and are expressed as total least-squares linear trends (°C) over the 25-year period extending from May 1963 to April 1988.

Hegerl et al. (1996)

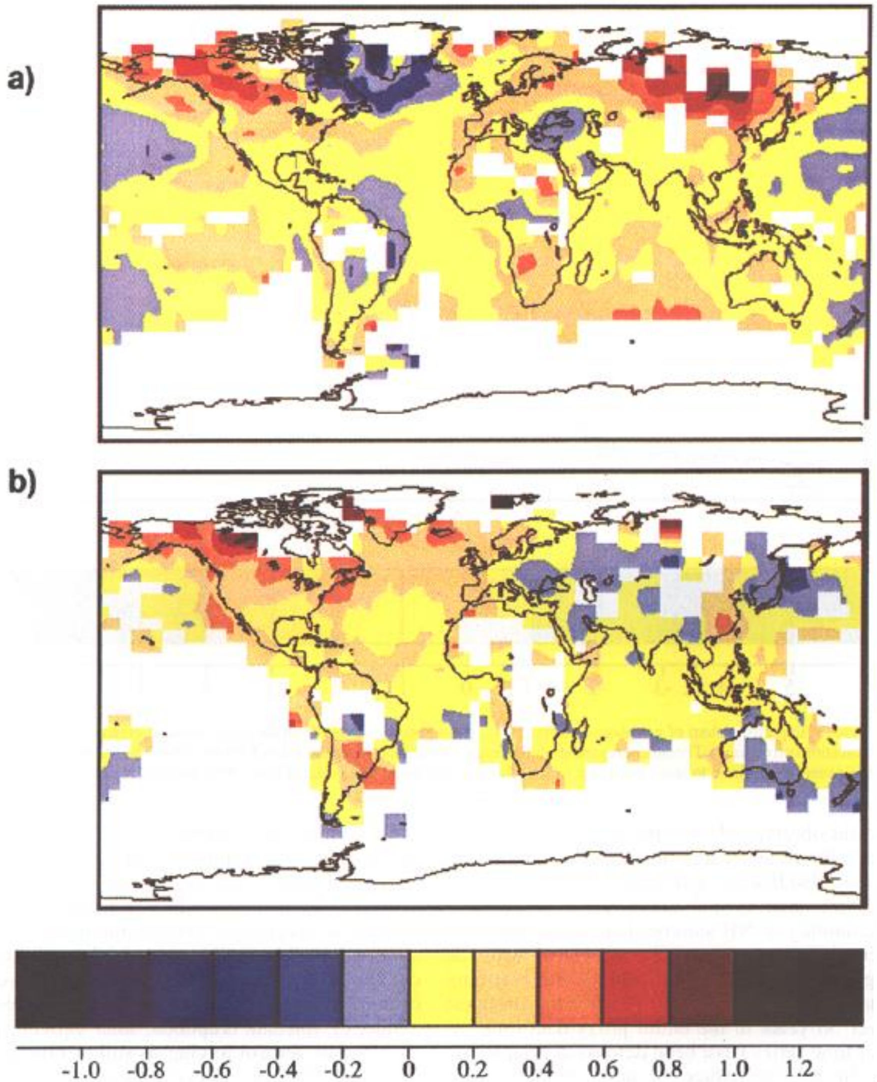


FIG. 2. Observed patterns of 30-yr trends for the periods 1965–1994 (a) and 1916–1945 (b) in degrees Celsius per decade, calculated from the data of Jones and Briffa (1992, 1994b).

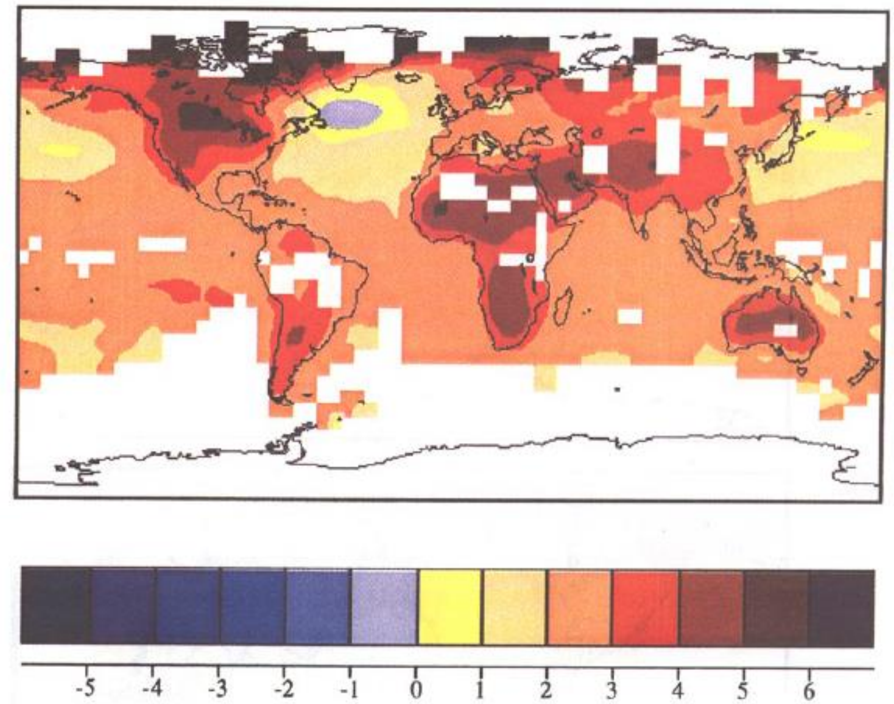


FIG. 3. Expected pattern of greenhouse warming (i.e., guess pattern) derived from the early industrial climate change (EIN) simulation (normalized). The pattern represents the difference between the decades 2076–2085 and 1986–1995 of the simulation. The pattern is restricted to space points for which reliable trends could be calculated since 1949 from the observations.

Alternative Formulation

- Levine and Berliner (*J. Clim* **12**, 564–574, 1999)
- Levine, Berliner and Shea (*J. Clim* **13**, 3805–3820, 2000)

Optimal Signal Detection

This is based on Levine's and Berliner's (*J. Clim* **12**, 564–574, 1999) reinterpretation of Hasselman's papers in *J. Clim* **6**, 1957–1971 (1993) and *Climate Dynamics* **13**, 601–611 (1997).

Suppose the observed climate signal $\mathbf{\Psi}$ satisfies

$$\mathbf{\Psi} \;=\; \mathbf{\Psi}^S + \tilde{\mathbf{\Psi}}$$

interpreted as "signal+noise". In practice, we usually assume both $\mathbf{\Psi}$ and $\mathbf{\Psi}^S$ are in fact *anomalies* from some reference time period. We also assume

- $\mathbf{\Psi}^S = \sum_{i=1}^{p} a_i \mathbf{g}_i$ where $\mathbf{g}_1, ..., \mathbf{g}_p$ are p known signal patterns and $a_1, ..., a_p$ are unknown weights; also write $\mathbf{\Psi}^S = G\mathbf{a}$.

- $\tilde{\mathbf{\Psi}}$ is a vector of "errors" with mean 0 and covariance matrix C.

Optimal Fingerprints (Hasselmann)

- $d_i = \mathbf{f}_i^T \mathbf{\Psi}$ is "detector" (and $\mathbf{f}_i$ is "fingerprint")

- $\mathbf{d}^S = (d_1^S, ..., d_p^S) = (\mathbf{f}_1^T \mathbf{\Psi}, ..., \mathbf{f}_p^T \mathbf{\Psi})$

- Fingerprints $\mathbf{f}_i$ constructed to maximize signal to noise ratio

$$\rho^2(\mathbf{d}^S) \;=\; (\mathbf{d}^S)^T D^{-1} \mathbf{d}^S,$$

  (i,j) entry of D is Cov$(\tilde{d}_i, \tilde{d}_j) =$ Cov$(\mathbf{f}_i^T \tilde{\Psi}, \mathbf{f}_j^T \tilde{\Psi}) = \mathbf{f}_i^T C \mathbf{f}_j$.

- This optimization problem leads to $\mathbf{f}_i^* = C^{-1} \mathbf{g}_i$, and hence $\mathbf{d}^* = G^T C^{-1} \mathbf{\Psi}$.

- Statistical significance of the signal determined through $\rho^2(\mathbf{d}^*)$.

Alternative Formulation (Levine & Berliner)

- Regression equation

$$\mathbf{\Psi} \;=\; G\mathbf{a} + \tilde{\mathbf{\Psi}}$$

- GLS estimates $\hat{\mathbf{a}} = (G^T C^{-1} G)^{-1} G^T C^{-1} \mathbf{\Psi}$ and hence $\hat{\mathbf{\Psi}}^S = G\hat{\mathbf{a}}$.

- Under Gaussian assumptions, $\hat{\mathbf{a}} \sim N[\mathbf{a}, (G^T C^{-1} G)^{-1}]$.

- Test $H_0 : \; \mathbf{a} = 0$ against $H_a : \; \mathbf{a} \neq 0$: UMPI test of level α rejects H_0 if

$$T \;=\; \mathbf{\Psi}^T C^{-1} G (G^T C^{-1} G)^{-1} G^T C^{-1} \mathbf{\Psi} \;>\; \chi_p^2(1-\alpha).$$

- But $T = \mathbf{\Psi}^T \mathbf{f}^* (G^T C^{-1} G)^{-1} (\mathbf{f}^*)^T \mathbf{\Psi} = \rho^2(\mathbf{d}^*)$. Therefore, the two tests are equivalent.

Other Issues

- Estimation of C

- They also considered the attribution question — estimates of $\mathbf{a}$, vector of coefficients of specified basis functions corresponding to known climate signals

- Null and alternative hypotheses the wrong way round? Analogy with *bioequivalence* problems

- Test is usually performed *only* if initial "detection" test rejects $\mathbf{a} = 0$. But that makes it harded to asses true significance level.

Bayesian Climate Change Assessment

Levine, Berliner and Shea (*J. Clim* **13**, 3805–3820, 2000)

Idea: Present and alternative Bayesian viewpoint of detection and attribution procedures

The approach of Myles Allen and collaborators

- Allen and Tett, "Checking for model consistency in optimal fingerprinting", *Climate Dynamics*, 1999

- Allen and Stott, "Estimating signal amplitudes on optimal fingerprinting, Part I: theory" *Climate Dynamics*, 2003

- Allen, Stott and Jones, "Estimating signal amplitudes on optimal fingerprinting, Part II: application to general circulation models" *Climate Dynamics*, 2003

- Huntingford *et al.*, "Incorporating model uncertainty into attribution of observed temperature change", *Geophysical Research Letters*, 2006

Allen and Tett (1999), Page 1

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

where

- $\mathbf{y}$ is vector of observations $(\ell \times 1)$
- $\mathbf{X}$ is matrix of m response patterns $(\ell \times m)$
- $\mathbf{u}$ is "climate noise", covariance matrix $\mathbf{C}$
- Assume normalizing matrix $\mathbf{P}$ such that $\mathbf{PCP}^T = \mathbf{I}$, $\mathbf{C}^{-1} = \mathbf{P}^T\mathbf{P}$.

Then

$$\mathbf{Py} = \mathbf{PX}\boldsymbol{\beta} + \mathbf{Pu}$$

where noise $\mathbf{Pu}$ has covariance matrix $\mathbf{I}$.

Allen and Tett (1999), Page 2

Then the Gauss-Markov Theorem implies

$$\tilde{\boldsymbol{\beta}} \;=\; (\mathbf{X}^T\mathbf{P}^T\mathbf{P}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{P}^T\mathbf{P}\mathbf{y}$$

with covariance matrix

$$V(\tilde{\boldsymbol{\beta}}) \;=\; (\mathbf{X}^T\mathbf{C}^{-1}\mathbf{X})^{-1}.$$

Confidence ellipsoid:

$$(\tilde{\boldsymbol{\beta}}-\boldsymbol{\beta})^T(\mathbf{X}^T\mathbf{C}^{-1}\mathbf{X})^{-1}(\tilde{\boldsymbol{\beta}}-\boldsymbol{\beta}) \sim \chi^2_m.$$

Main difficulty: $\mathbf{C}$ is unknown.

We could have a have a vector of n independent "noise" simulations $\mathbf{y}_N$ and then estimate $\hat{\mathbf{C}} = \frac{1}{n}Y_N Y_N^T$ but typically $n << \ell$ so $\hat{\mathbf{C}}$ is singular.

Allen and Tett (1999), Page 3

Resolution:

- Restrict to κ EOFs with largest variance (equivalent to replacing $\mathbf{P}$ by $\mathbf{P}^\kappa$, consisting of the κ eigenvectors of $\mathbf{C}$ with largest eigenvalues).
- *Independent* control runs used to estimate $\mathbf{C}$

Have an estimate $\tilde{V}(\tilde{\boldsymbol{\beta}})$ with ν degrees of freedom (*also needs to be estimated because of autocorrelation in series of control runs*). Then:

$$(\tilde{\boldsymbol{\beta}} - \boldsymbol{\beta})^T \tilde{V}(\tilde{\boldsymbol{\beta}})^{-1}(\tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}) \sim mF_{m,\nu}$$

Allen and Tett (1999), Page 4

Final theoretical step: testing the fit

Define

$$\tilde{\mathbf{u}} \;=\; \mathbf{y} - \mathbf{X}\tilde{\boldsymbol{\beta}}.$$

Then

$$r^2 \;=\; \tilde{\mathbf{u}}^T \mathbf{C}^{-1} \tilde{\mathbf{u}} \;\sim\; \chi^2_{\kappa - m}.$$

With independent control runs

$$\tilde{\mathbf{u}}^T \widehat{\mathbf{C}}^{-1} \tilde{\mathbf{u}} \;\sim\; (\kappa - m) F_{\kappa - m, \nu}.$$

This can be used as a diagnostic on the model fit and also to guide the choice of κ.

Total Least Squares

Basic equation still

$$Y = \sum_{j=1}^{m} \beta_j X_j + \eta$$

where Y is the observational record (e.g. a vector of trend in temperature means), $X_1, ..., X_m$ are the signals from m climate models, and η is an error term.

Instead of ordinary least squares, Allen and Stott (2003) proposed to fit (1) by *total least squares*, which allows for errors in the X_j's as well as Y. Technique extended by Huntingford, Stott, Allen and Lambert (2006).

The motivation is that, in practice, the X's are also unknown.

We reformulate this based on a classical (non-Bayesian) treatment of the errors in variables problem (Gleser 1981).

Single x variable (Allen & Stott)

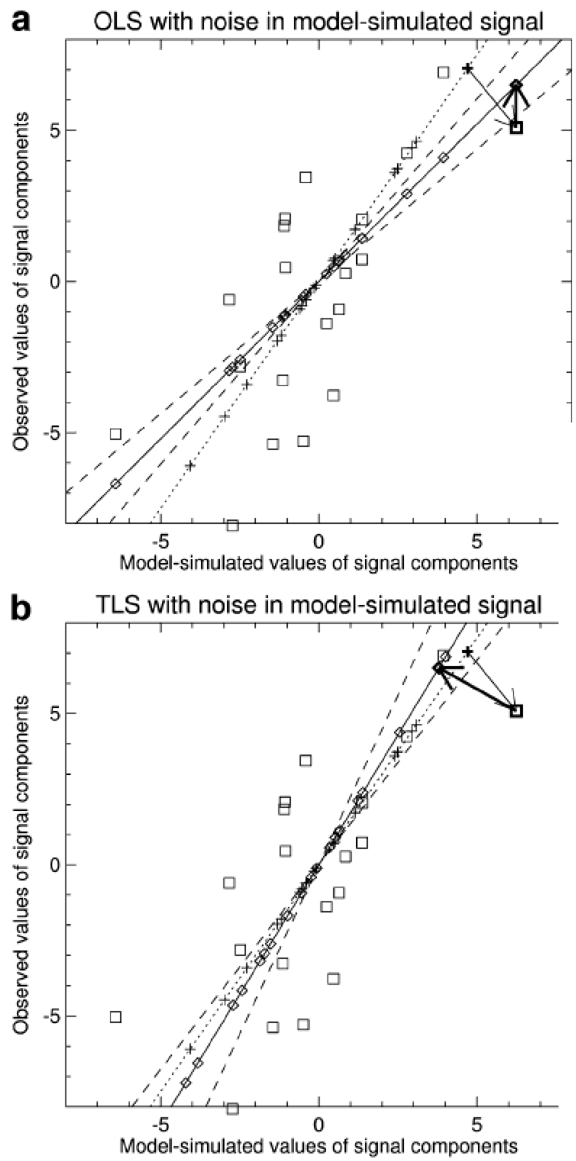


Fig. 2 **a**: application of ordinary least squares regression to a system in which both "model" (*plotted in the horizontal*) and "observations" (*plotted in the vertical*) are contaminated with equal levels of noise. "True" values (normally unobservable, except this is a synthetic example, and uncontaminated with any noise) are plotted as *crosses* along the *dotted line*; noise-contaminated "observations" and "simulation" are plotted as *squares*, with the *thin arrow* showing the orientation of the noise vector in one case; best-fit line and reconstructed observations are shown as the *diamonds*, with *heavy arrow* showing the hypothetical noise that is minimised in the OLS algorithm. The best estimate is biased towards zero under OLS and, in this example, the 5–95% confidence interval, shown by the *dashed lines*, does not include the correct slope. **b**: application of total least squares regression to the same example. TLS minimises the perpendicular distance from the best-fit line, shown by the *heavy arrow*, not the vertical distance minimised by OLS. The bias towards zero slope is removed, and the 5–95% confidence interval on the slope now includes the correct value

Multiple x Variables

Allen and Stott considered the multiple regression extension of this — key assumption is the same noise structure in each covariate *and* in the control model runs

$$\mathbf{y} = \sum_{i=1}^{m} (\mathbf{x}_i - \boldsymbol{\nu}_i)\beta_i + \boldsymbol{\nu}_0$$

where each $\boldsymbol{\nu}_i$ has the same noise structure as $\boldsymbol{\nu}_0$.

Motivation for this assumption: we don't actually know the noise structure of either $\boldsymbol{\nu}_0$ or $\boldsymbol{\nu}_i,\ i \geq 1$, but the only tool we have to estimate either is the set of control runs, so we assume the same covariances for control runs as for model simulations that include forcing factors.

To paraphrase Allen and Stott, a method that even crudely accounts for the variances of the X variables is surely better than one that ignored these issues altogether.

Gleser's formulation of errors in variables (EIV)

Ref: L.J. Gleser, *Annals of Statistics*, 1981

$$x_i = \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix} = \begin{pmatrix} u_{i1} \\ u_{i2} \end{pmatrix} + \begin{pmatrix} e_{i1} \\ e_{i2} \end{pmatrix}, \tag{1}$$

x_{i1} and x_{i2} observations of dimensions p and r respectively, u_{i1} and u_{i2} are true unobserved signals, e_{i1} and e_{i2} noise with covariances $\sigma^2 I_p$ and $\sigma^2 I_r$. Also

$$u_{i2} = Bu_{i1}. \tag{2}$$

MLE: choose B and $u_{i1}, ..., u_{in}$ to minimize

$$Q = \frac{1}{2\sigma^2}\sum_i (x_{i1} - u_{i1})^T (x_{i1} - u_{i1}) + \frac{1}{2\sigma^2}\sum_i (x_{i2} - Bu_{i1})^T (x_{i2} - Bu_{i1}). \tag{3}$$

Computing the MLE

Choose B to minimize

$$\tilde{Q} \;=\; \frac{1}{2\sigma^2}\sum_i (x_{i2} - Bx_{i1})^T (I_r + BB^T)^{-1}(x_{i2} - Bx_{i1}). \qquad (4)$$

(also called *generalized least squares* by Gleser)

Asymptotic Distribution Theory (Gleser)

Assume estimator $\hat{B}_n$ based on n observations, U_1 the $p \times n$ matrix whose columns are $u_{i1}, ..., u_{in}$.

[Assumption A:] e_i are i.i.d. random vectors with mean 0 and common covariance matrix $\sigma^2 I_{p+r}$

[Assumption C:] $\Delta = \lim_{n\to\infty} n^{-1} U_1 U_1^T$ exists, positive definite.

[Assumption E:] The cross-moments of the common distribution of the e_i are identical, up to and including moments of order 4, to the corresponding moments of the multivariate normal distribution with the same mean and covariance matrix.

Then the elements of the $n^{1/2}(\hat{B} - B)$ have an asymptotic rp-dimensional normal distribution with mean 0 and the covariance between the (i, j) and (i', j') elements is given by $\sigma^2[\sigma^2 \Delta^{-1}(I_p + B^T B)^{-1} \Delta^{-1} + \Delta^{-1}]_{jj'} \cdot [I_r + BB^T]_{ii'}$.

Application to Allen-Stott Model

Identify x_{i2} with Y (single observation, dimension r)

Identify x_{i1} with $(X_1, ..., X_m)^T$ (dimension rm). Also

$$B = \left(\beta_1 I_r \;\; \beta_2 I_r \;\; \ldots \;\; \beta_m I_r \right). \tag{5}$$

GLSE chooses $\beta_1, ..., \beta_m$ to minimize

$$S = \frac{(Y - \sum_j \beta_j X_j)^T (Y - \sum_j \beta_j X_j)}{1 + \sum \beta_j^2}. \tag{6}$$

Equivalent to Allen and Stott (2003). In principle, we could use Gleser's theory to approximate the asymptotic (co-)variances of the estimators, though this is problematic because $n = 1$...

Huntingford, Stott, Allen and Lambert(2006)

Previously

$$\mathbf{y} = \sum_{i=1}^{m} (\mathbf{x}_i - \boldsymbol{\nu}_i)\beta_i + \boldsymbol{\nu}_0 \qquad (7)$$

Now assume each signal is derived as a mean $\bar{\mathbf{x}}_i$ over several models and rewrite (7) as

$$\mathbf{y} = \sum_{i=1}^{m} (\bar{\mathbf{x}}_i - \boldsymbol{\nu}_i - \boldsymbol{\mu}_i)\beta_i + \boldsymbol{\nu}_0 \qquad (8)$$

where $\boldsymbol{\mu}_i$ is intended to capture the uncertainty of model projections for signal i.

Assume different noise structure for $\boldsymbol{\mu}_i$ as for $\boldsymbol{\nu}_i$, uses Bayesian EIV method from Nounou *et al.*, *AIChE J.* (2002) ("Bayesian latent variable regression")

More Recent Developments

- A. Ribes, J.-M. Azaïs and S. Planton (2009), Adaptation of the optimal fingerprint method for climate change detection using a well-conditioned covaroance matrix estimate. *Climate Dynamics*

- A. Ribes, S. Planton and L. Terray (2013), Application of regularised optimal fingerprinting to attribution. Part I: Method, properties and idealised analysis. *Climate Dynamics*

- A. Hannart, A. Ribes and P. Naveau (2014), Optimal fingerprinting under multiple sources of uncertainty. *Geophysical Research Letters*

- A. Hannart (2016), Integrated optimal fingerprinting: Method description and illustration. *Journal of Climate*

- M. Katzfuss, D. Hammerling and R. Smith (2017), A Bayesian hierarchical model for climate change detection and attribution. *Geophysical Research Letters*

The Model of Katzfuss–Hammerling–Smith (1)

Consider the basic structure

$$\mathbf{y} \mid \mathbf{x}_1, ..., \mathbf{x}_M, \boldsymbol{\beta}, \mathbf{C}, \alpha \;\sim\; \mathcal{N}_n \left\{ \sum_{m=1}^{M} \boldsymbol{\beta}_j \mathbf{x}_j, \mathbf{C} \right\}. \tag{9}$$

where

- $\mathbf{y}$ is the vector of "true" temperatures ($n \times 1$);
- $\mathbf{x}_m$ for $1 \le m \le M$ is the "signal" for the m'th forcing factor;
- $\boldsymbol{\beta}$ is the vector of regression coefficients ($M \times 1$);
- $\mathbf{C}$ is the covariance matrix representing internal variability;

The Model of Katzfuss–Hammerling–Smith (2)

The observational error equation is

$$\mathbf{y}^{(i)} \mid \mathbf{y}, \mathbf{W} \;\sim\; \mathcal{N}_n(\mathbf{y}, \mathbf{W}),\; i = 1, ..., N, \tag{10}$$

for N independent reconstructions of $\mathbf{y}$ with assumed covariance matrix $\mathbf{W} = \mathrm{diag}(\omega_1, ..., \omega_n)$.

The model error equation is

$$\mathbf{x}_m^{(\ell)} \mid \mathbf{x}_m, \mathbf{C} \;\sim\; \mathcal{N}_n(\mathbf{x}_m, \mathbf{C}),\; \ell = 1, ..., L_m,\; m = 0, ..., M. \tag{11}$$

Most of the remaining slides are taken from a presentation prepared by Dorit Hammerling (National Center for Atmospheric Research), based on the paper by Katzfuss, Hammerling and Smith (2017, *Geophysical Research Letters*)

Bayesian D&A regression model

Bayesian regression model:

$$\mathbf{y}|\mathbf{X}, \boldsymbol{\beta}, \mathbf{C} \sim N_n\Big(\sum_{j=1}^{m} \beta_j \mathbf{x}_j, \mathbf{C} \Big)$$

D&A consists of determining the posterior distribution of the β_j (mainly, $P(\beta_j > 0|\mathbf{y}, \mathbf{X})$).

Challenge: $\mathbf{y}$, $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_m)$, $\boldsymbol{\beta}$, and $\mathbf{C}$ are all unknown

NCAR NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

Uncertainty in observed temperature changes

True temperature changes in grid cells over the globe are unknown
But: We have an ensemble of N temperature time series, which can be converted to an ensemble of N temperature changes

We assume that

$$\mathbf{y}^{(i)}|\mathbf{y}, \mathbf{W} \overset{iid}{\sim} N_n(\mathbf{y}, \mathbf{W}), \quad i = 1, \ldots, N,$$

where $\mathbf{W}$ is a covariance matrix describing the variability of the ensemble members around the true temperature change

NCAR
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

Uncertainty in temperature under forcing scenarios

The (true) temperature changes due to forcing are also unknown, but we have an ensemble of GCM outputs for each forcing scenario:

$$\mathbf{x}_j^{(l)} | \mathbf{x}_j, \mathbf{C} \overset{ind}{\sim} N_n(\mathbf{x}_j, \mathbf{C}), \quad l = 1, \ldots, L_j, \; j = 1, \ldots, m,$$

where L_j is the number is the number of GCM runs under the jth forcing scenario, and climate variability is assumed to have covariance matrix $\mathbf{C}$.

Model parameters

- Climate variability: Typically, $\mathbf{C}$ is expanded in *empirical orthogonal functions* and then truncated:
  $\mathbf{C} = \mathbf{BKB}'$, where $\mathbf{B}$ contains the first r principal components estimated from control runs, $\mathbf{K} = diag\{e^{\lambda_1}, \ldots, e^{\lambda_r}\}$, and $r << n$
- Observation uncertainty: Currently, $\mathbf{W} = \sigma^2 \widetilde{\mathbf{W}}$, where $\widetilde{\mathbf{W}}$ is a diagonal matrix containing the empirical variances of $\{\mathbf{y}^{(i)}\}$
- Priors:
  - Noninformative priors for β and σ
  - Vaguely informative priors for $\lambda_1, \ldots, \lambda_r$

Inference

MCMC with adaptive Metropolis-Hastings updates

High-dimensional problem $\rightarrow$ Integrate out $\mathbf{y}$ and $\mathbf{X}$

MCMC computations only rely on low-dimensional quantities and are very fast, even for almost a million data points

Bayesian model averaging

Previous slides assumed r, the number of EOFs, to be fixed.

The number of variables in the model depends on $r \rightarrow$ standard MCMC sampler cannot be used to make inference on θ and r simultaneously.

Instead we perform Bayesian model averaging (BMA) to average the posterior results for each value of r using weights automatically chosen by the data.

NCAR
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

Bayesian model averaging (cont.)

The posterior of β averaged over the posterior of r (i.e., taking the uncertainty about the value of r into account) is given by

$$[\beta|\mathcal{Y},\mathcal{X}] = \sum_{i=\min}^{\max} [\beta|r_i,\mathcal{Y},\mathcal{X}][r_i|\mathcal{Y},\mathcal{X}],$$

Due to the uniform prior on r, the posterior probability of $r = r_i$ is given by $[r_i|\mathcal{Y},\mathcal{X}] \propto [\mathcal{Y}|r_i,\mathcal{X}]\,[r_i] \propto [\mathcal{Y}|r_i,\mathcal{X}]$.

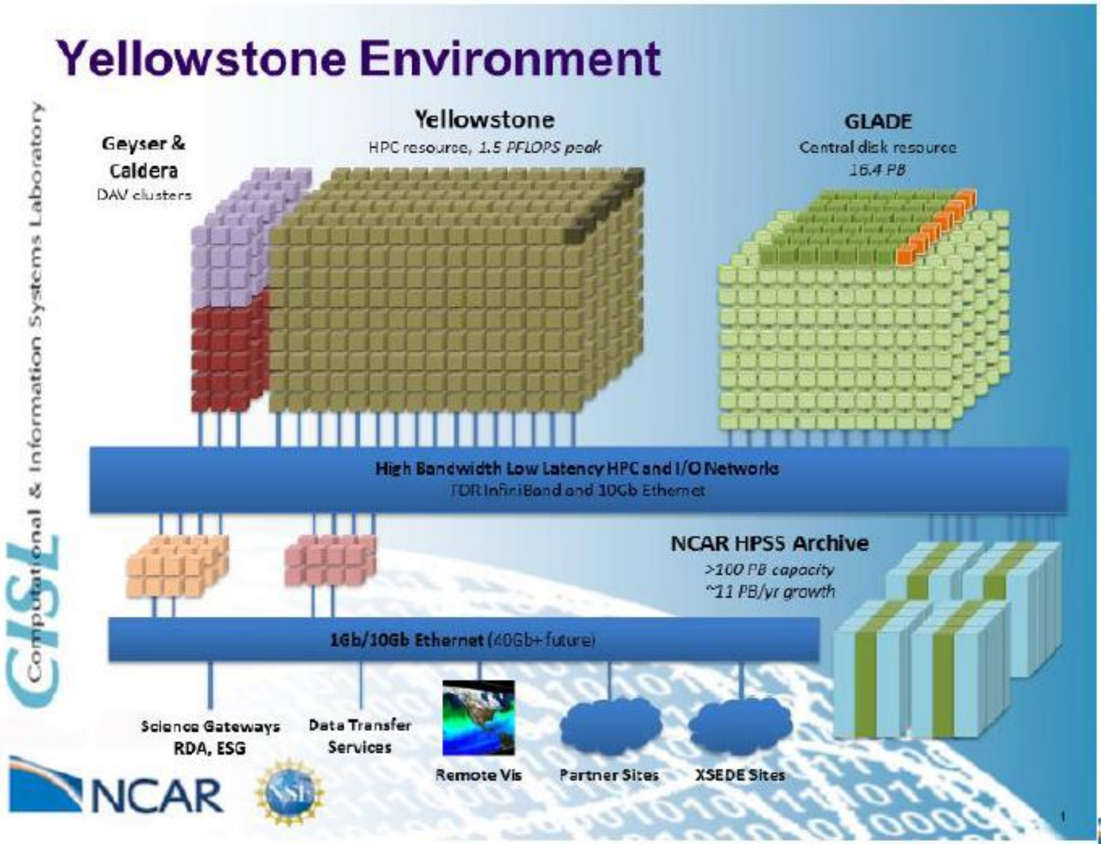
Fortunately, a good estimate of marginal likelihood $[\mathcal{Y}|r_i,\mathcal{X}]$ can be obtained using the evaluations of the likelihood already performed in the MCMC procedure as

$$[\mathcal{Y}|r_i,\mathcal{X}] = \frac{1}{M}\sum_{j=1}^{M}[\mathcal{Y}|r_i,\boldsymbol{\theta}^{(j)},\mathcal{X}][\boldsymbol{\theta}^{(j)}|r_i]$$

NCAR
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

Computational considerations

Bayesian modeling averaging approach is ideally suited for parallelization.



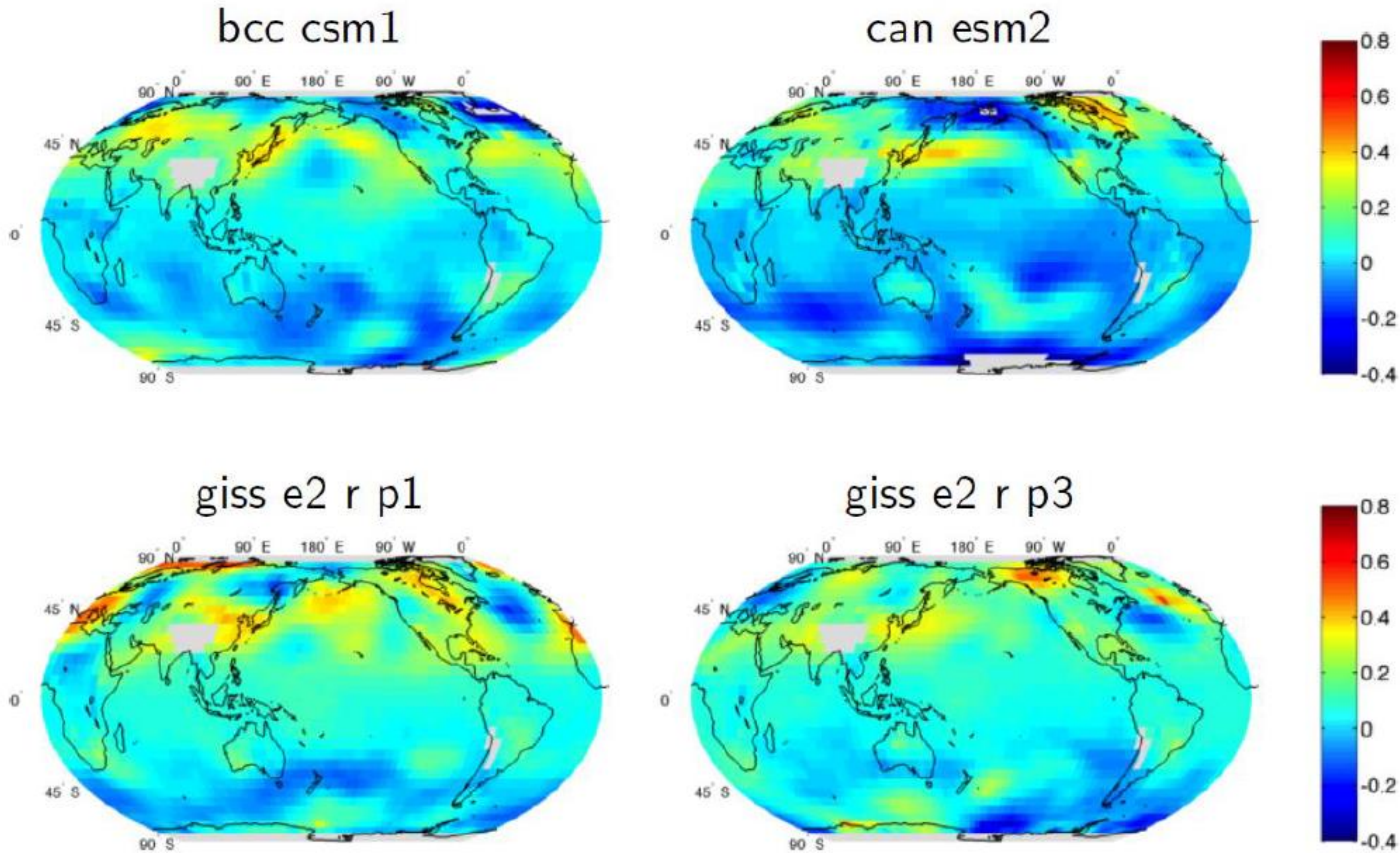
Parallelizing over r (161EOFs): 40 hours on laptop → 2 hours on Geyser

The data

- Climate Model Intercomparison project (CMIP5) models: suite of more than 20 models, of which we use as subset (BCC CSM1, CAN ESM2, CSIRO, GISS, IPSL, GFDL)
- Remote Sensing Systems temperature retrievals based on microwave sounding units (MSUs): $N = 396$ realizations

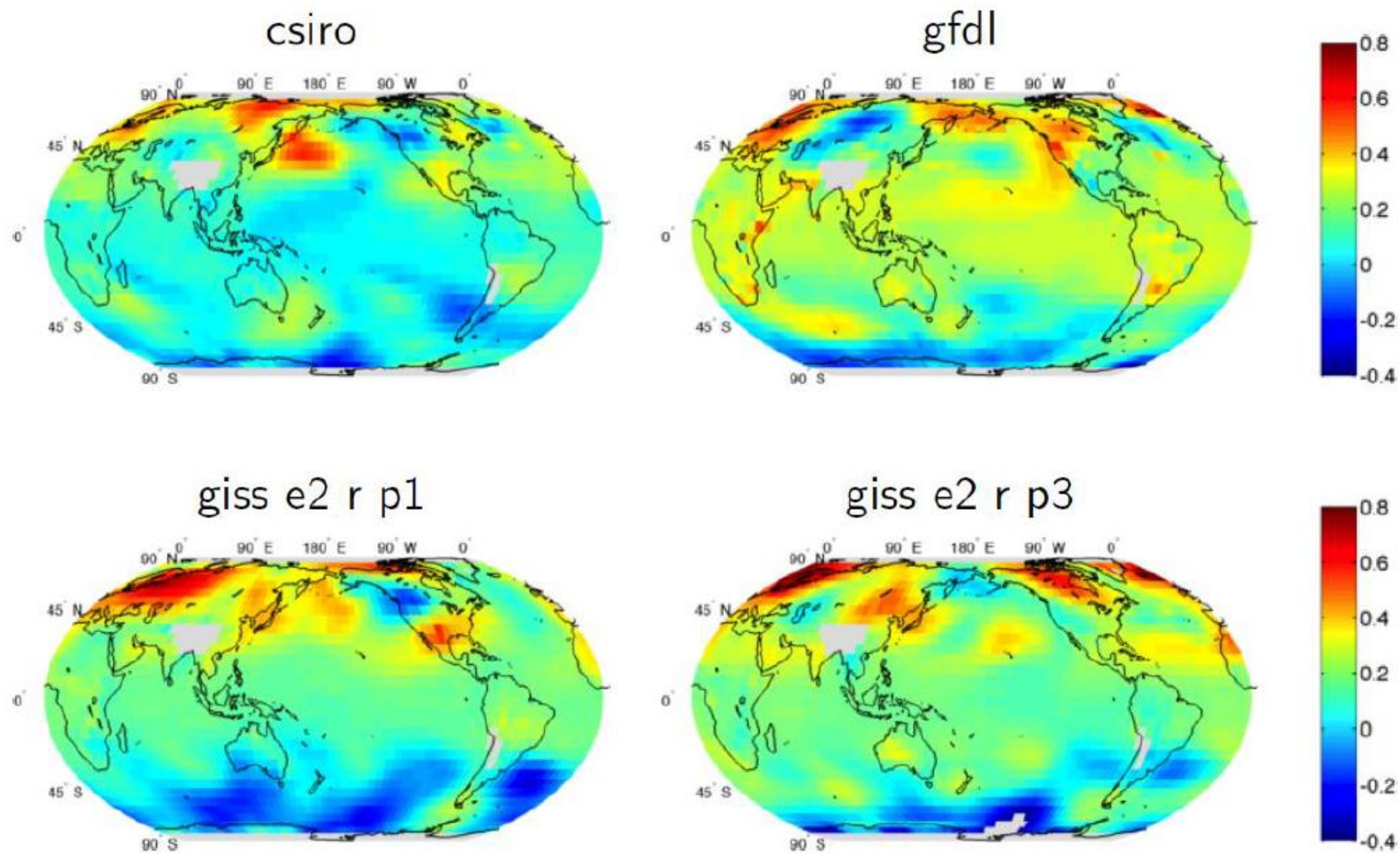
Based on these sources, we consider the linear trends (slopes) of annual lower-tropospheric temperatures between 1979 and 2005 in $n = 2107$ $5^\circ \times 5^\circ$ grid cells on the globe (between -70° and 80° latitude with an altitude lower than 3km)

Linear trends 1979–2005: Natural-only forcing



Units are °C per decade

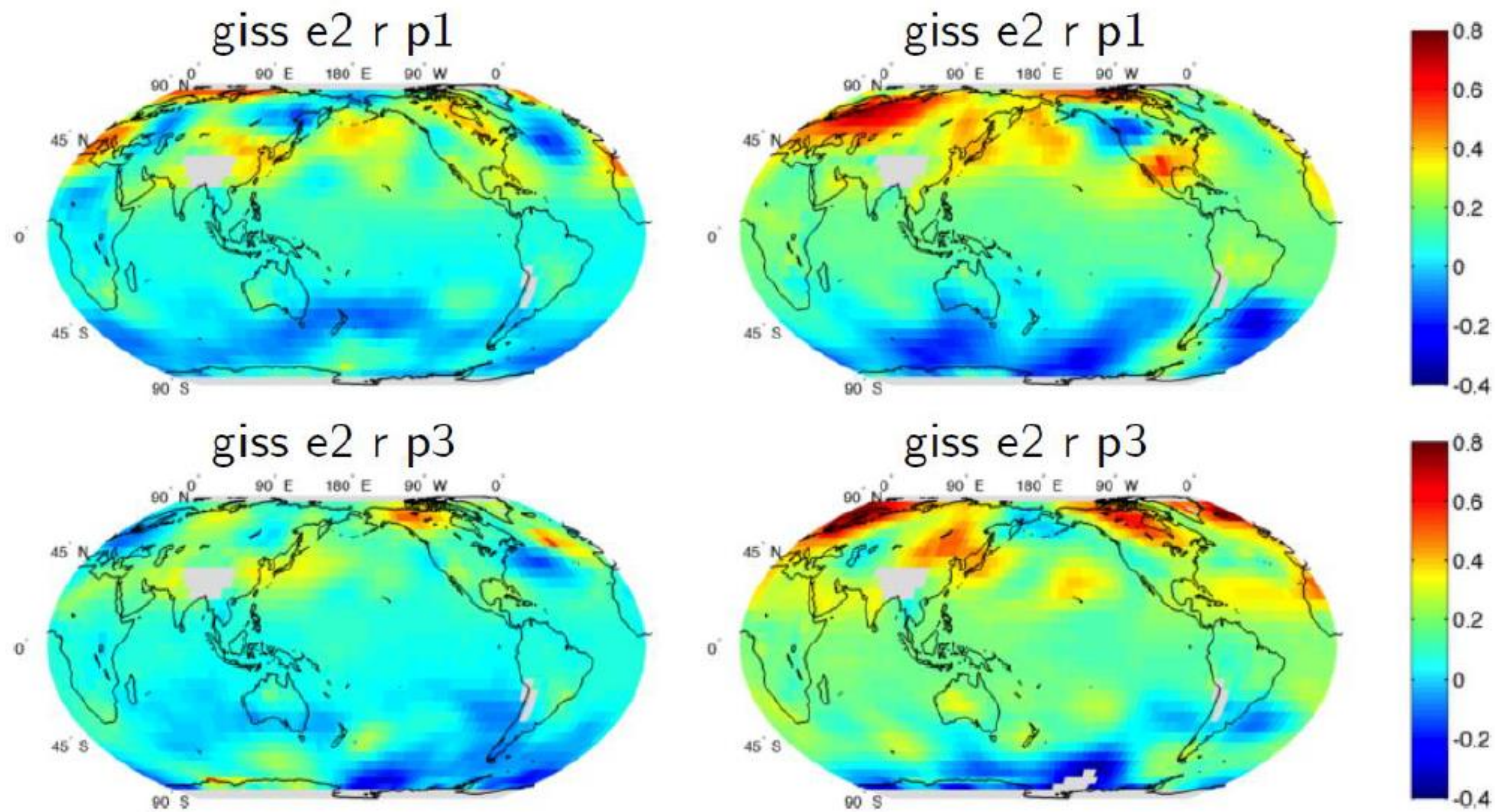
Linear trends 1979–2005: Anthropogenic-only forcing



Units are °C per decade

Linear trends 1979–2005: giss models

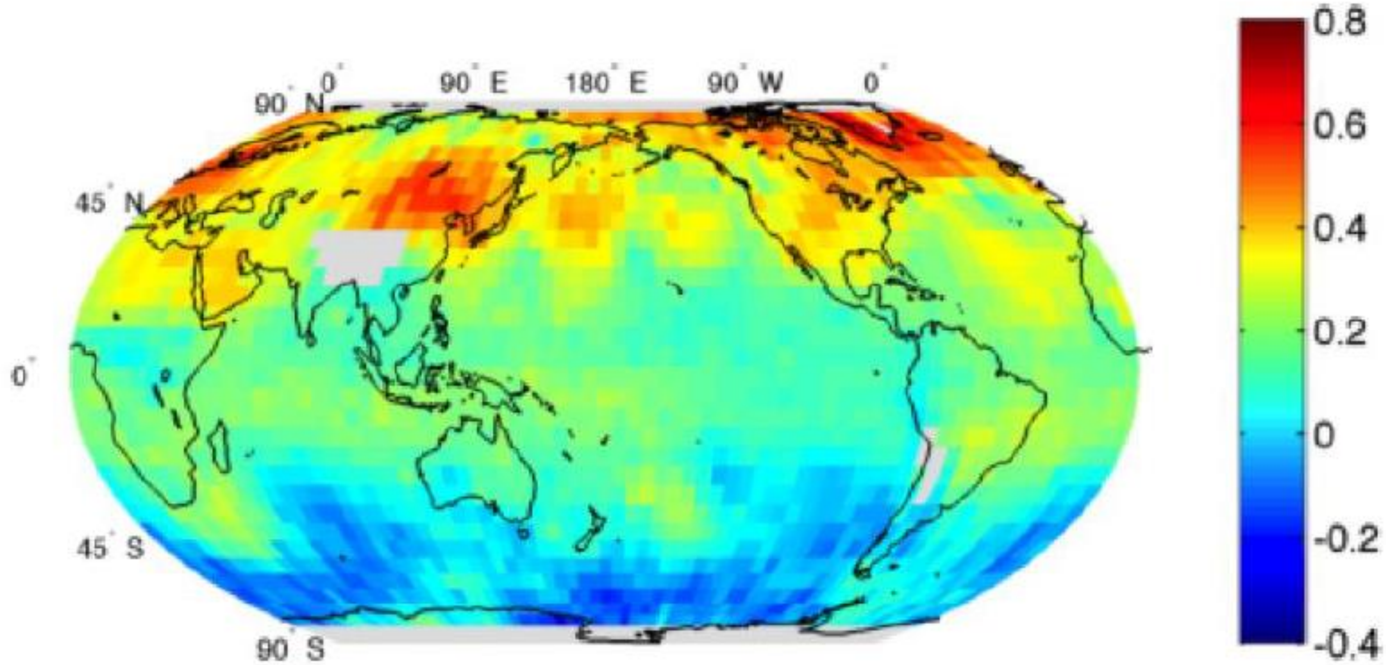
Natural-only forcing | Anthropogenic-only forcing



Units are °C per decade

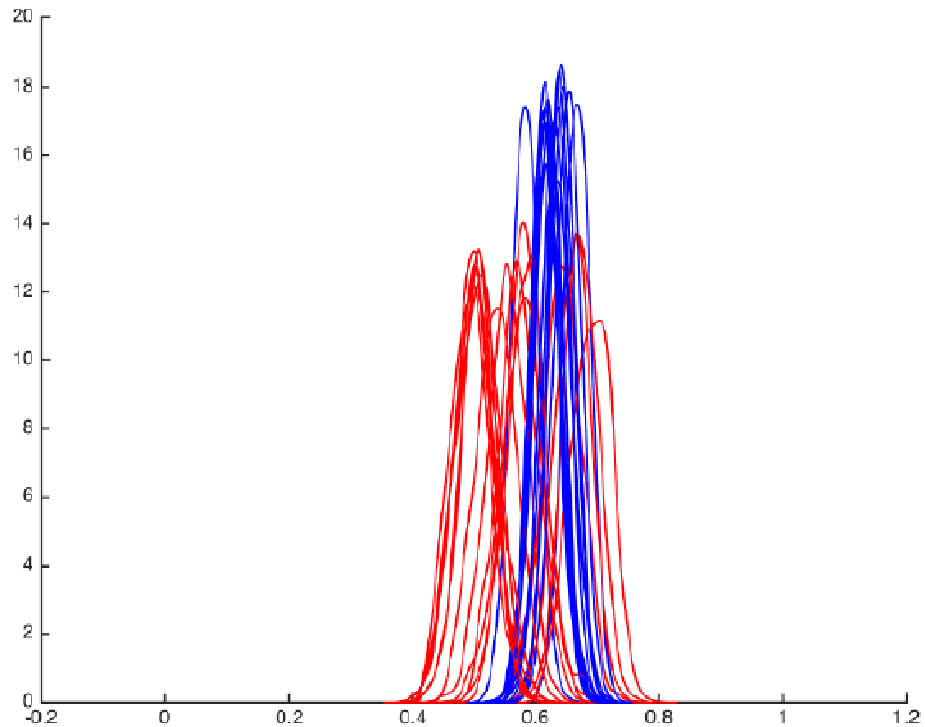
Linear trends in satellite observations 1979–2005

Average of 396 ensemble members



Posterior densities for βs for all values of r

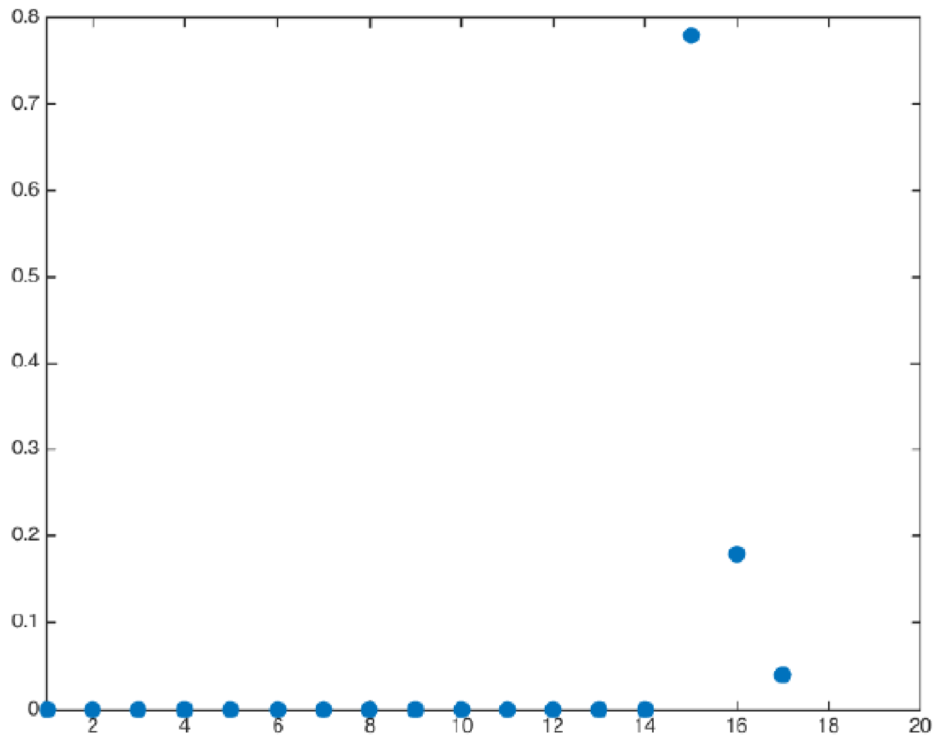
All available GCM models for forced runs, bcc model for control (18 runs)



blue(β_1) corresponds to anthropogenic forcings, red(β_2) to natural forcings

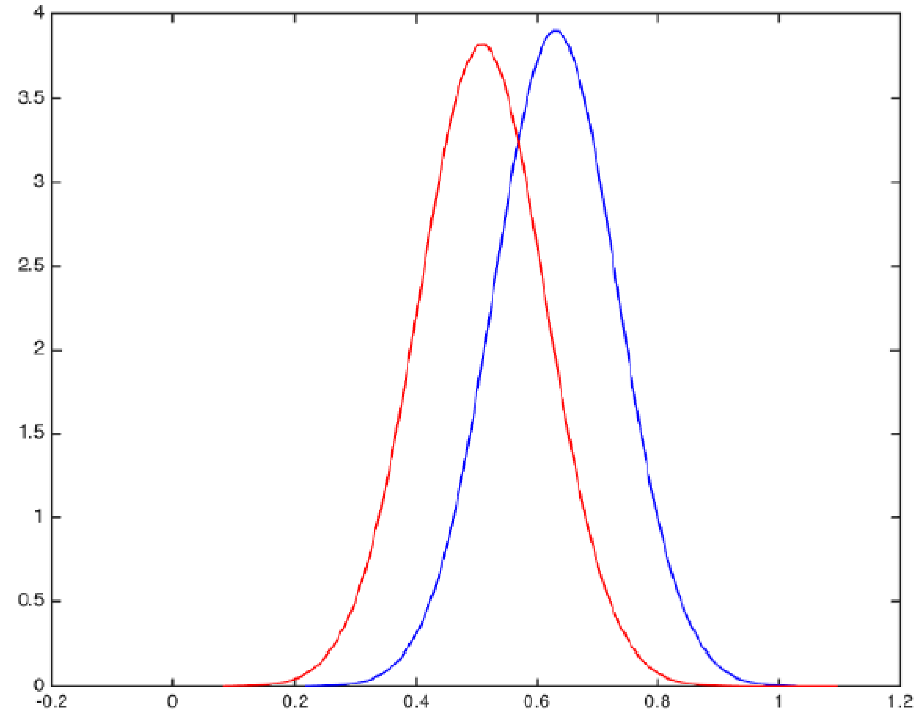
Weights for all values of r

All available GCM models for forced runs, bcc model for control (18 runs)



Bayesian model averaged posterior densities for βs

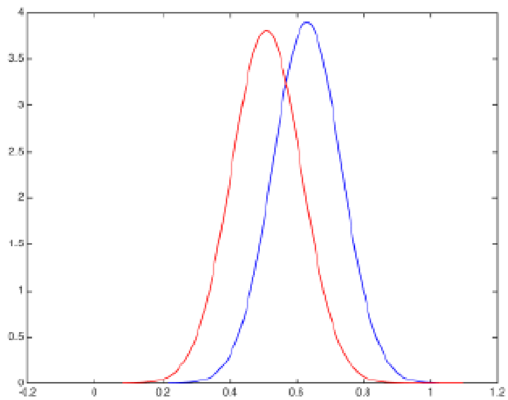
All available GCM models for forced runs, bcc model for control (18 runs)



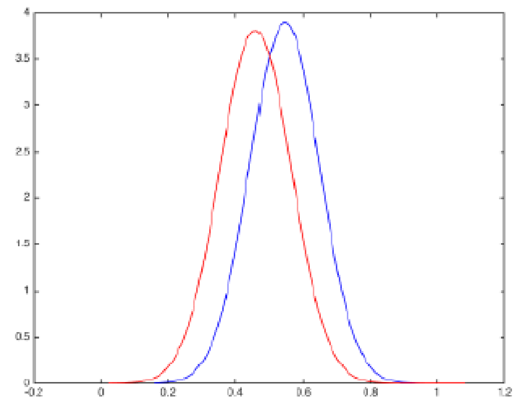
blue(β_1) corresponds to anthropogenic forcings, red(β_2) to natural forcings

Posterior densities using different control runs

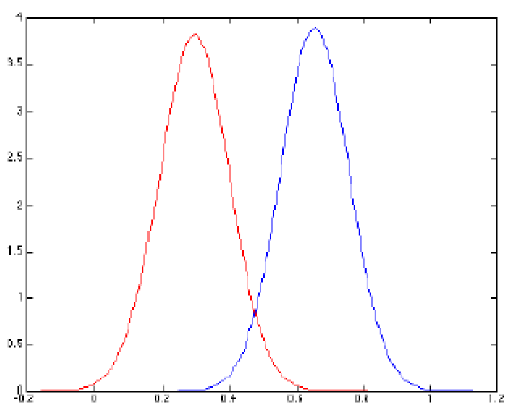
Control runs: bcc



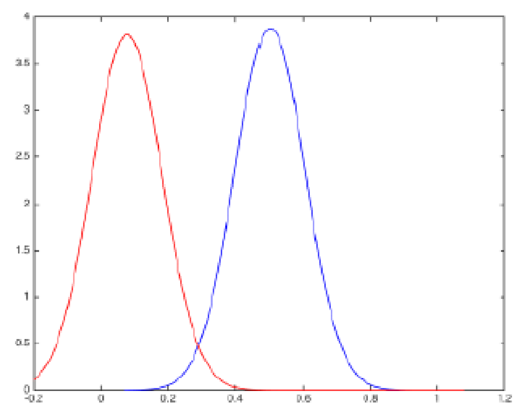
Control runs: gfdl



Control runs: csiro + ipsl



Control runs: from nine models



Relation to Method of Hannart (J. Climate 2016)

Hannart presented an "integrated Optimal Fingerprinting" approach that has several overlaps with the current method

- Not expicitly Bayesian but uses several elements derived from Bayesian theory, in particular, *integrated likelihoods*
- Initial $\widehat{C} = S$ where S is sample covariance matrix from control model runs
- Improved estimate $\widehat{C}_\alpha = \alpha\Delta + (1-\alpha)S$ for suitably chosen α, Δ
- Inverse Wishart "prior distribution" on C; integrate out C from Likelihood
- Didn't take account of observational uncertainty
- Open question which method performs better

Summary

- BHM allows natural modeling of uncertainty in all quantities in the D&A regression model
- Posteriors take all (modeled) uncertainties into account
- Results not sensitive to priors
- BUT results are sensitive to choice of control runs

Future work:

- Inference on EOFs themselves
- Or completely different approach to estimating covariance

For another viewpoint on this whole subject, I recommend the video by Aurélien Ribes from BIRS

https://www.birs.ca/events/2016/5-day-workshops/16w5092/videos

PRESENTED BY:

Dr. Kerry Emanuel

Professor of Atmospheric Science

MIT

SAMSI Public Lecture

The Storm Next Time: Hurricanes and Climate Change

Monday, October 9, 2017 @ 7:30pm

Genome Sciences Building, G100 Auditorium

University of North Carolina - Chapel Hill

The recent tragedy of Hurricane Harvey, together with earlier extreme events such as Hurricanes Katrina and Sandy, has raised the question whether the apparent increasing severity of such events can be attributed to the human influence on greenhouse gas warming. Dr. Emanuel will review the growing consensus that the incidence of the strongest storms will increase over time, even though there may be a decline of the far more numerous weaker events.

*****This lecture is free and open to the public!***