STATISTICS FOR CLIMATE SCIENCE

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University of North Carolina and SAMSI

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Kolkata, March 2-4, 2015

www.unc.edu/~rls/kolkata.html

THE UNIVERSITY *of* NORTH CAROLINA *at* CHAPEL HILL

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In Memoriam
Gopinath Kallianpur 1925 - 2015

I. TIME SERIES ANALYSIS FOR CLIMATE DATA

I.a Overview

I.b The post-1998 “hiatus” in temperature trends

I.c NOAA’s record “streak”

I.d Trends or nonstationarity?

II. CLIMATE EXTREMES

II.a Extreme value models

II.b An example based on track records

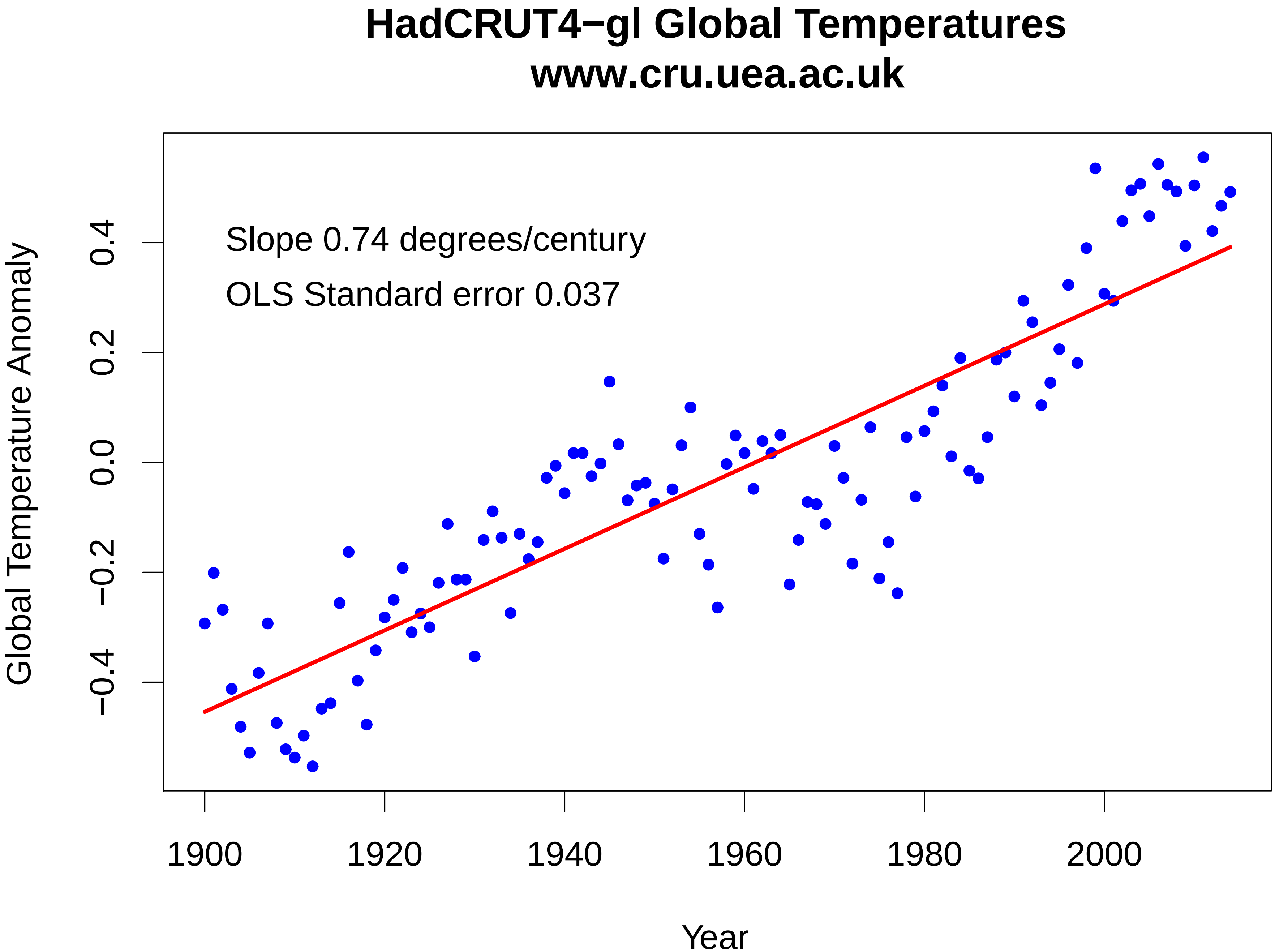
II.c Applying extreme value models to weather extremes

II.d Joint distributions of two of more variables

II.e Conclusions, other models, future research

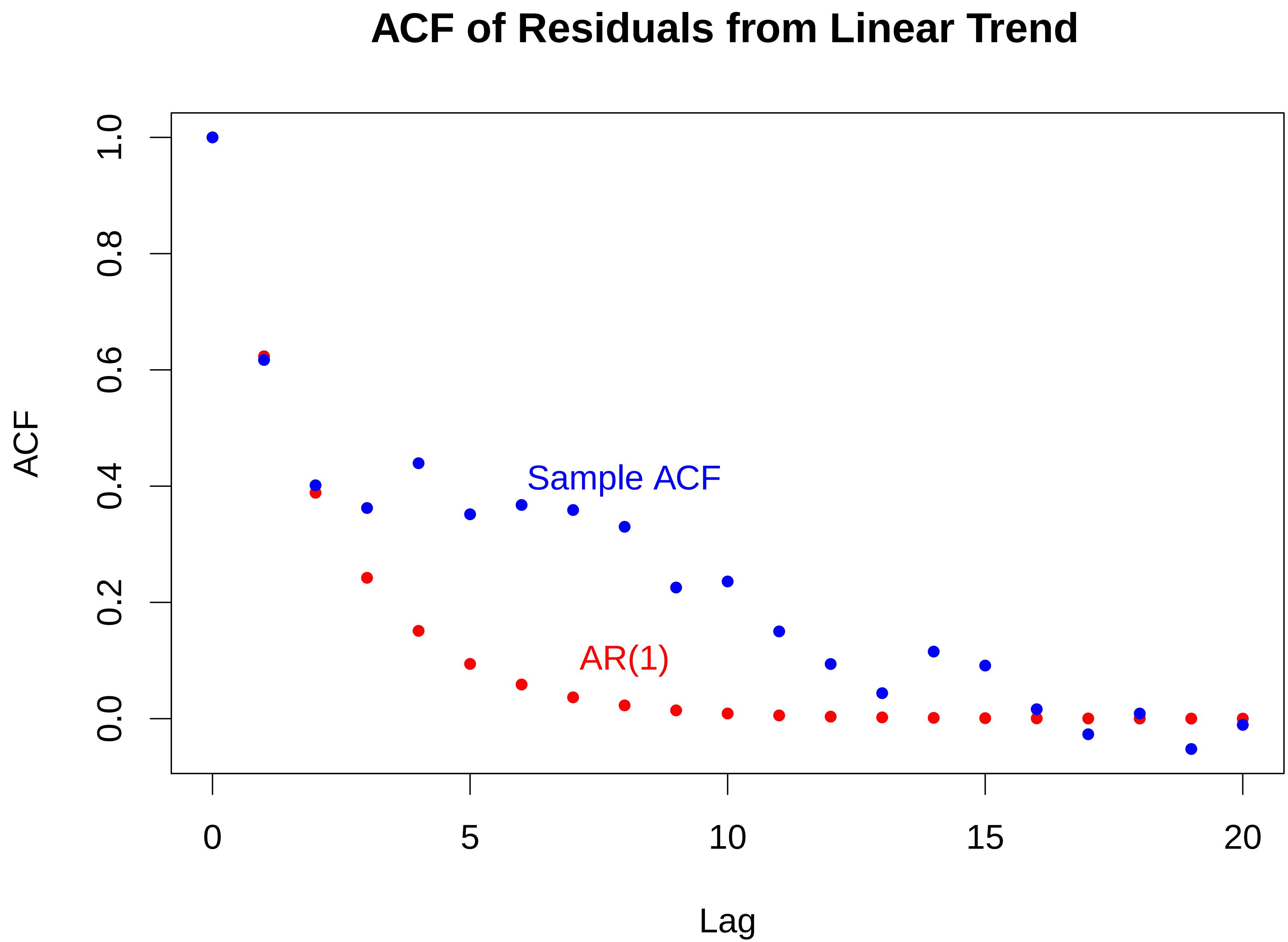
I. TIME SERIES ANALYSIS FOR CLIMATE DATA

I.a Overview


HadCRUT4−gl Global Temperatures
www.cru.uea.ac.uk
Slope 0.74 degrees/century
OLS Standard error 0.037
Global Temperature Anomaly
−0.4
−0.2
0.0
0.2
0.4
1900
1920
1940
1960
1980
2000
Year

What's wrong with that picture?

- We fitted a linear trend to data which are obviously autocorrelated
- OLS estimate 0.74 deg C per century, standard error 0.037
- So it looks statistically significant, but question how standard error is affected by the autocorrelation
- First and simplest correction to this: assume an AR(1) time series model for the residual
- So I calculated the residuals from the linear trend and fitted an AR(1) model, $X_n = \phi_1 X_{n-1} + \epsilon_n$, estimated $\widehat{\phi}_1 = 0.62$ with standard error 0.07. *With this model*, the standard error of the OLS linear trend becomes 0.057, still making the trend very highly significant
- *But is this an adequate model?*


ACF of Residuals from Linear Trend
ACF
1.0
0.8
0.6
0.4
0.2
0.0
Sample ACF
AR(1)
0
5
10
15
20
Lag

Fit AR(p) of various orders p, calculate log likelihood, AIC, and the standard error of the linear trend.

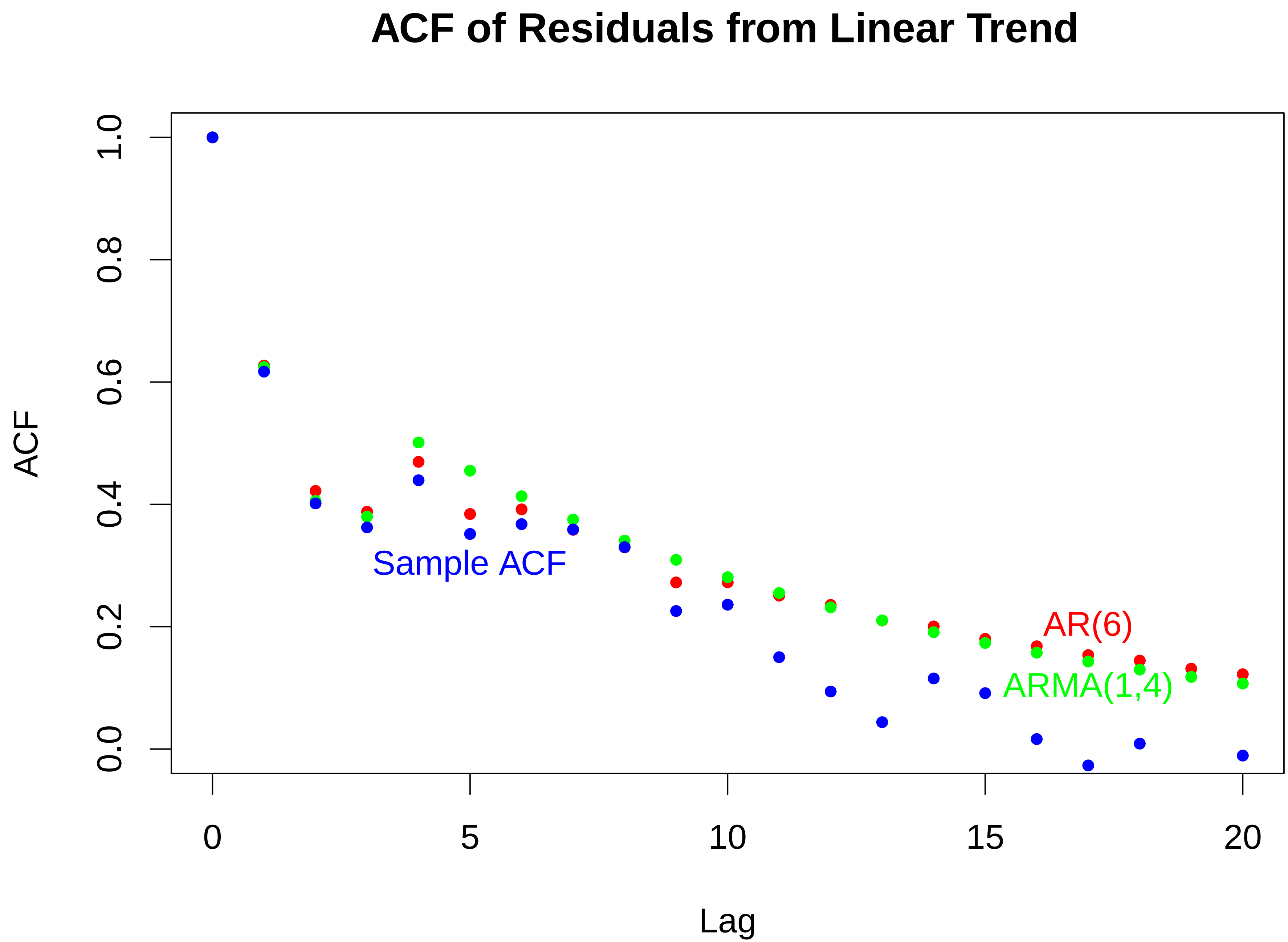
Model $X_n = \sum_{i=1}^{p} \phi_i X_{n-i} + \epsilon_n, \ \epsilon_n \sim N[0, \sigma_\epsilon^2]$ (IID)

AR order	LogLik	AIC	Trend SE
0	72.00548	−140.0110	0.036
1	99.99997	−193.9999	0.057
2	100.13509	−192.2702	0.060
3	101.84946	−193.6989	0.069
4	105.92796	−199.8559	0.082
5	106.12261	−198.2452	0.079
6	107.98867	−199.9773	0.086
7	108.16547	−198.3309	0.089
8	108.16548	−196.3310	0.089
9	108.41251	−194.8250	0.086
10	108.48379	−192.9676	0.087

Extend the calculation to ARMA(p,q) for various p and q: model is $X_n - \sum_{i=1}^{p} \phi_i X_{n-i} = \epsilon_n + \sum_{j=1}^{q} \theta_j \epsilon_{n-j}$, $\epsilon_n \sim N[0, \sigma_\epsilon^2]$ (IID)

AR order	MA order					
	0	1	2	3	4	5
0	−140.0	−177.2	−188.4	−186.4	−191.5	−192.0
1	−194.0	−193.0	−197.4	−195.5	−201.7	−199.8
2	−192.3	−193.0	−195.4	−199.2	−200.8	−199.1
3	−193.7	−197.2	−200.3	−197.9	−200.8	−200.1
4	−199.9	−199.6	−199.8	−197.8	−196.8	−197.4
5	−198.2	−198.8	−197.8	−195.8	−194.8	−192.8
6	−200.0	−198.3	−196.4	−195.7	−196.5	−199.6
7	−198.3	−196.3	−200.2	−199.1	−194.6	−197.3
8	−196.3	−195.8	−194.4	−192.5	−192.8	−196.4
9	−194.8	−194.4	−197.6	−197.9	−196.2	−194.4
10	−193.0	−192.5	−195.0	−191.2	−194.9	−192.4

SE of trend based on ARMA(1,4) model: 0.087 deg C per century


ACF of Residuals from Linear Trend
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Lag
Sample ACF
AR(6)
ARMA(1,4)

Calculating the standard error of the trend

Estimate $\widehat{\beta} = \sum_{i=1}^{n} w_i X_i$, variance

$$\sigma_\epsilon^2 \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \rho_{|i-j|}$$

where ρ is the autocorrelation function of the fitted ARMA model

Alternative formula (Bloomfield and Nychka, 1992)

$$\text{Variance}(\widehat{\beta}) \;=\; 2\int_0^{1/2} w(f)s(f)df$$

where $s(f)$ is the spectral density of the autocovariance function and

$$w(f) \;=\; \left| \sum_{j=1}^{n} w_n e^{-2\pi i j f} \right|^2$$

is the *transfer function*

What's better than the OLS linear trend estimator?

Use *generalized least squares* (GLS)

$$
\begin{aligned}
y_n &= \beta_0 + \beta_1 x_n + u_n, \\
u_n &\sim ARMA(p,q)
\end{aligned}
$$

Repeat same process with AIC: ARMA(1,4) again best

$\widehat{\beta} = 0.73$, standard error 0.10.

Calculations in R

```
ip=4
iq=1
ts1=arima(y2,order=c(ip,0,iq),xreg=1:ny,method='ML')
```

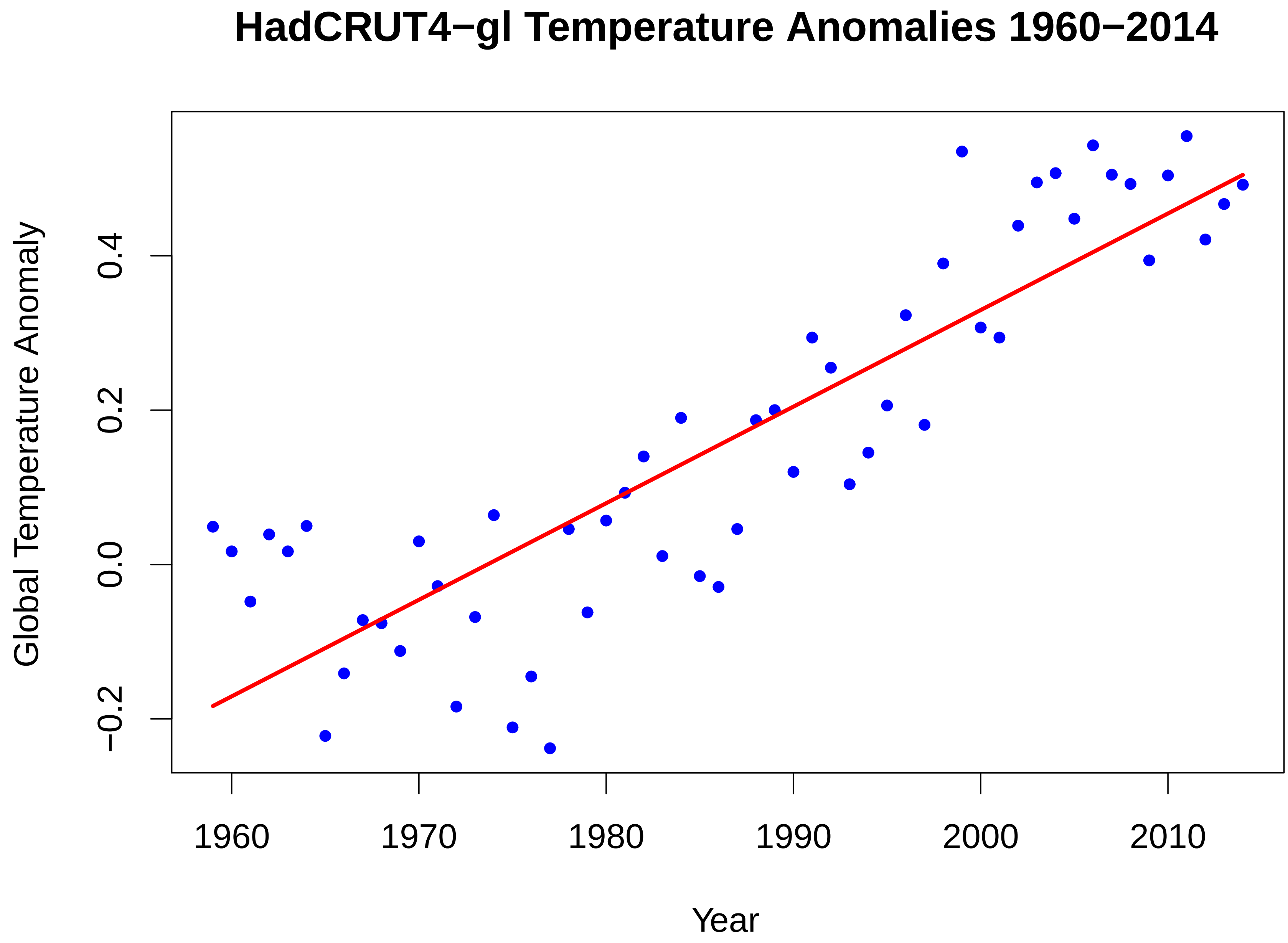
```
Coefficients:
         ar1     ar2     ar3     ar4     ma1  intercept    1:ny
      0.0058  0.2764  0.0101  0.3313  0.5884    -0.4415  0.0073
s.e.  0.3458  0.2173  0.0919  0.0891  0.3791     0.0681  0.0010
sigma^2 estimated as 0.009061:  log likelihood = 106.8,
aic = -197.59
```

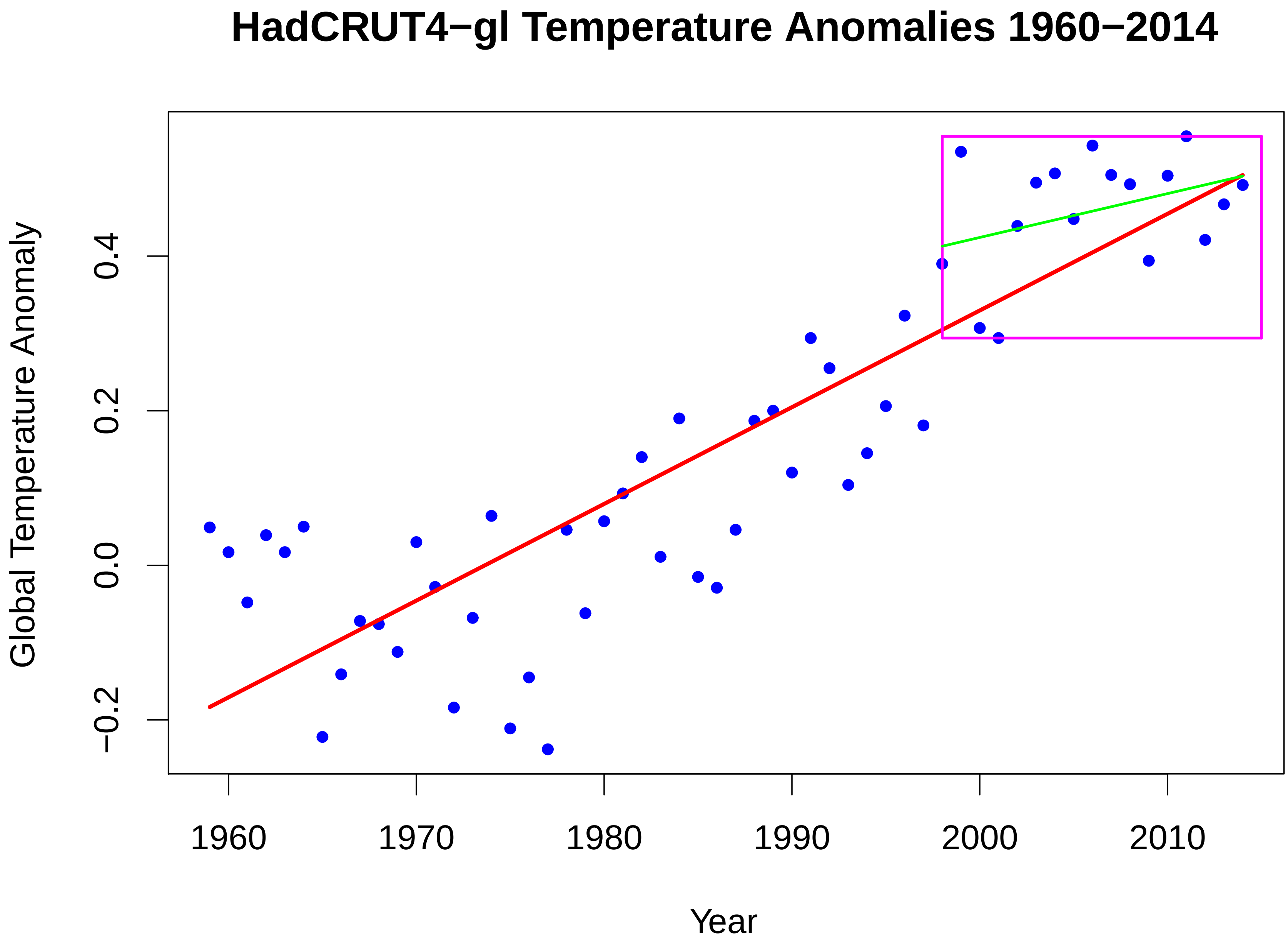
```
acf1=ARMAacf(ar=ts1$coef[1:ip],ma=ts1$coef[ip+1:iq],lag.max=150)
```

I. TIME SERIES ANALYSIS FOR CLIMATE DATA

I.a Overview

I.b The post-1998 “hiatus” in temperature trends


HadCRUT4−gl Temperature Anomalies 1960−2014
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
1960
1970
1980
1990
2000
2010
Year


HadCRUT4−gl Temperature Anomalies 1960−2014
Global Temperature Anomaly
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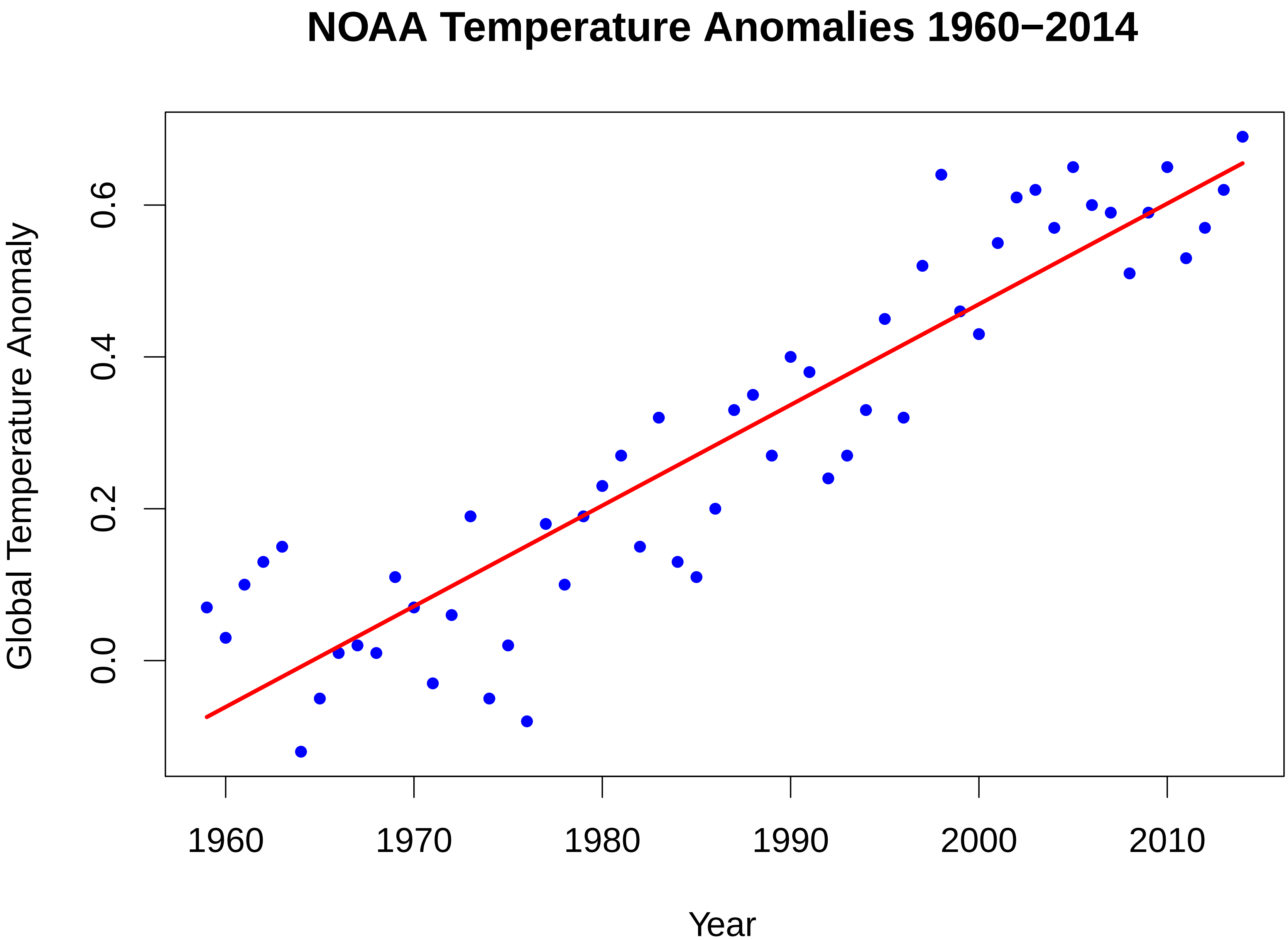
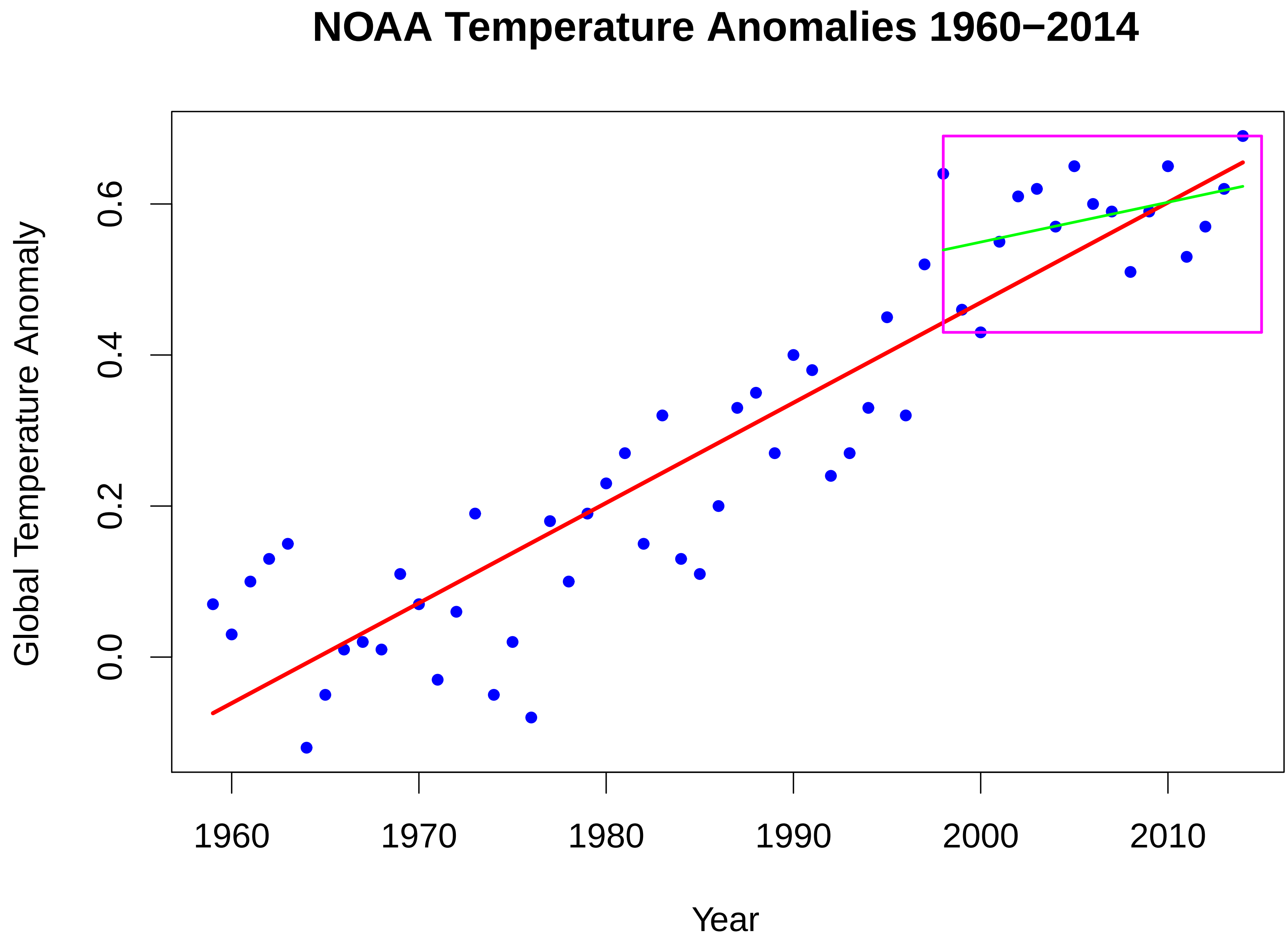
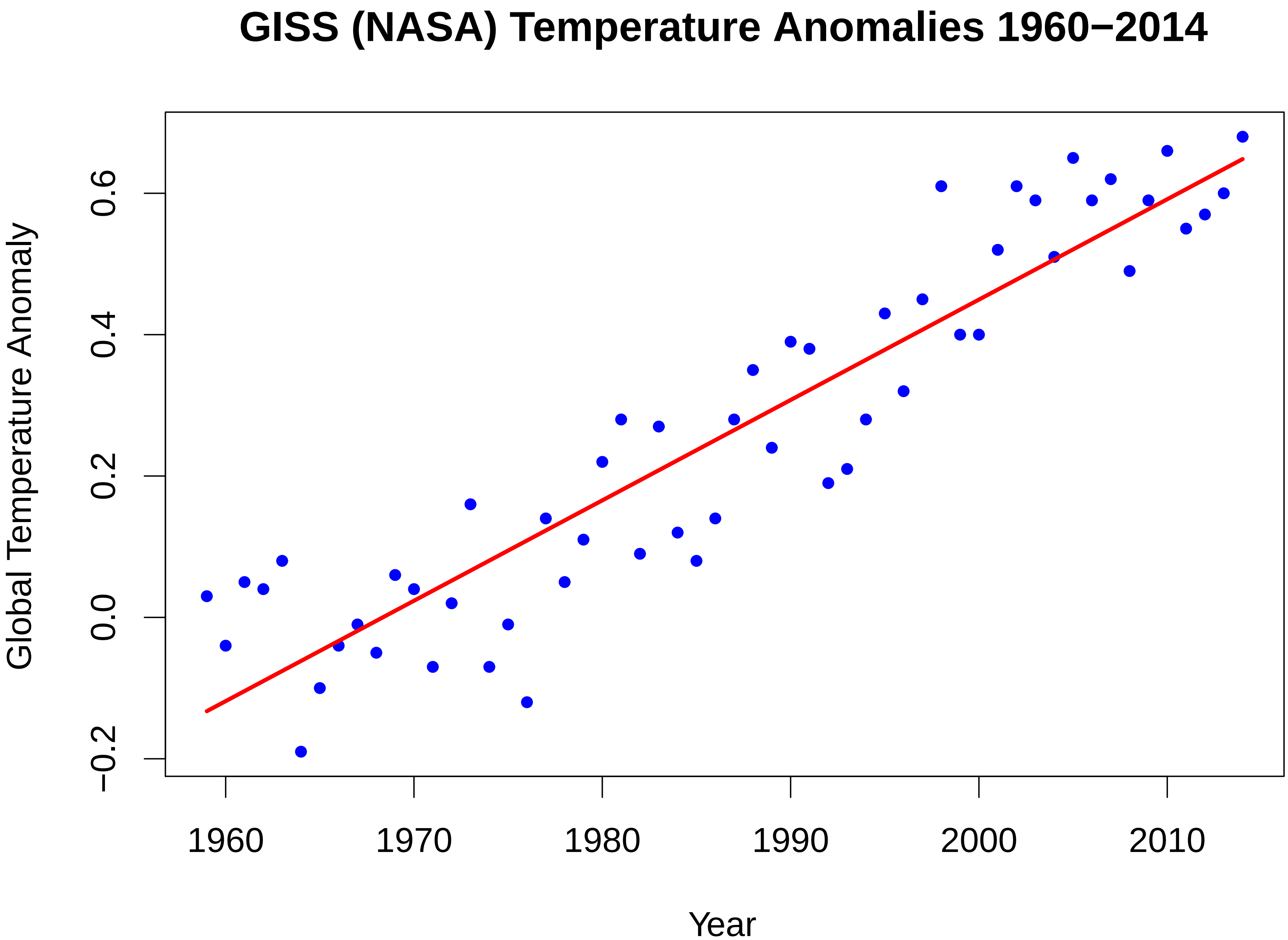
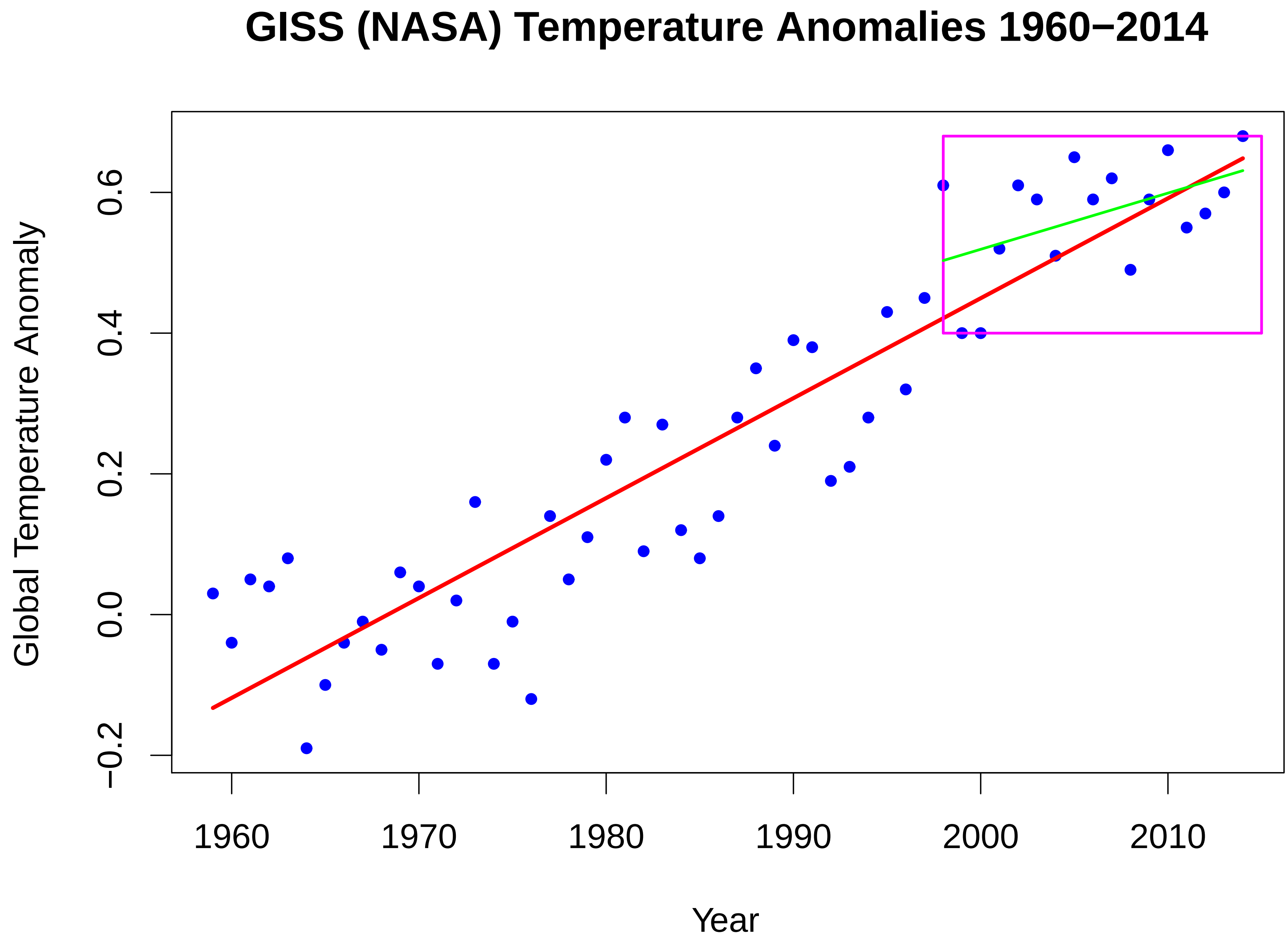
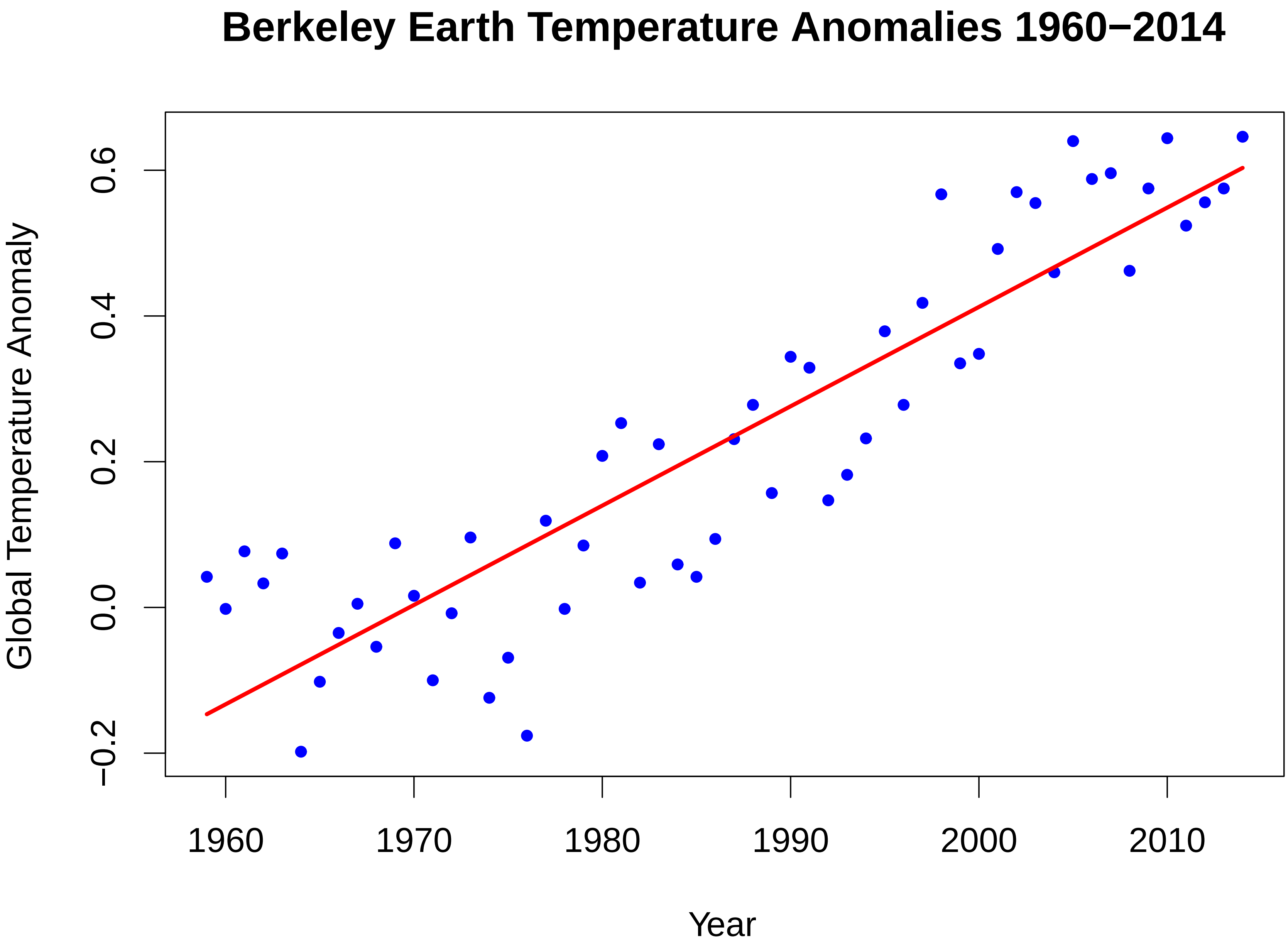
2010
Year

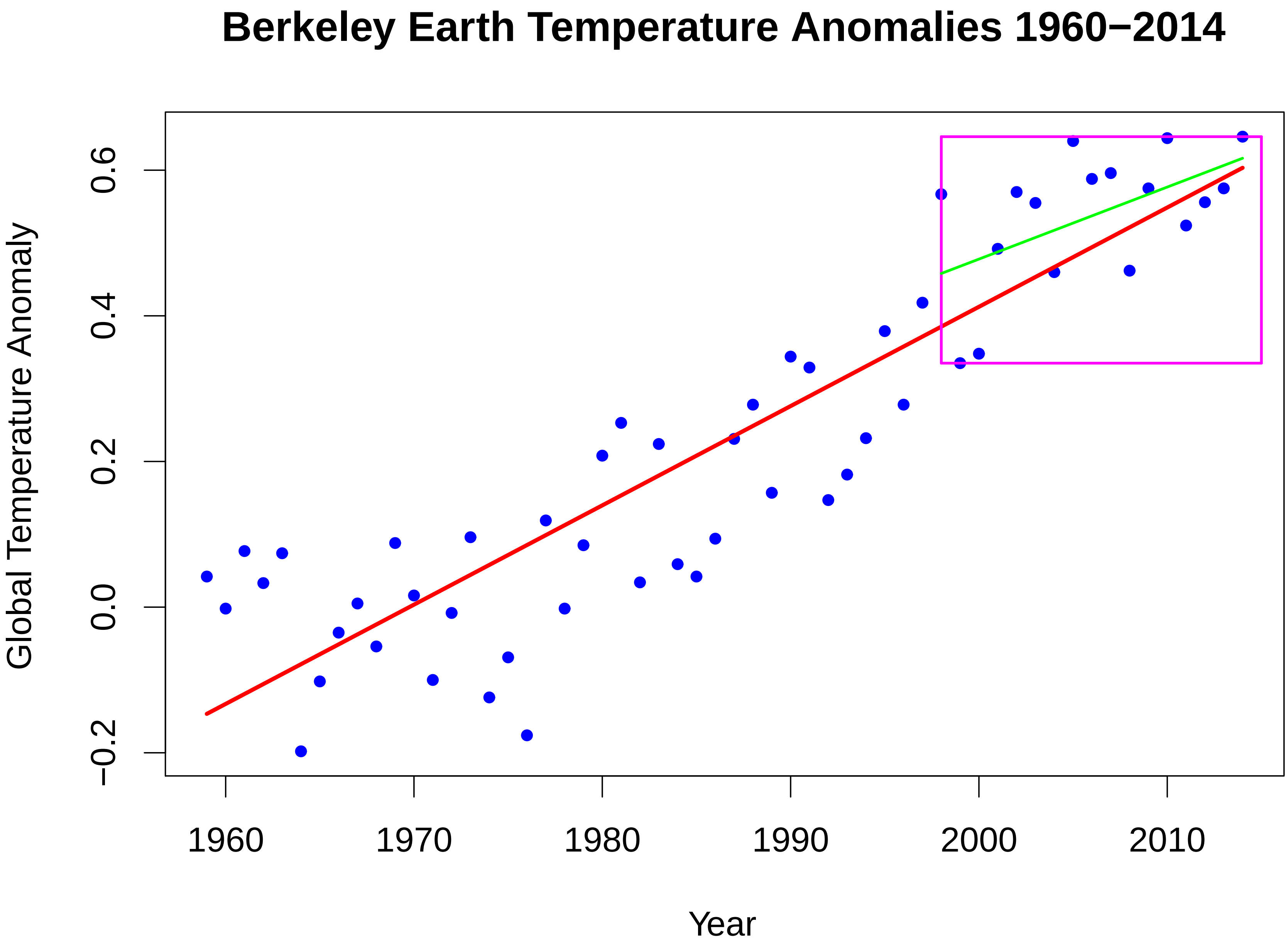

NOAA Temperature Anomalies 1960−2014
Global Temperature Anomaly
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year

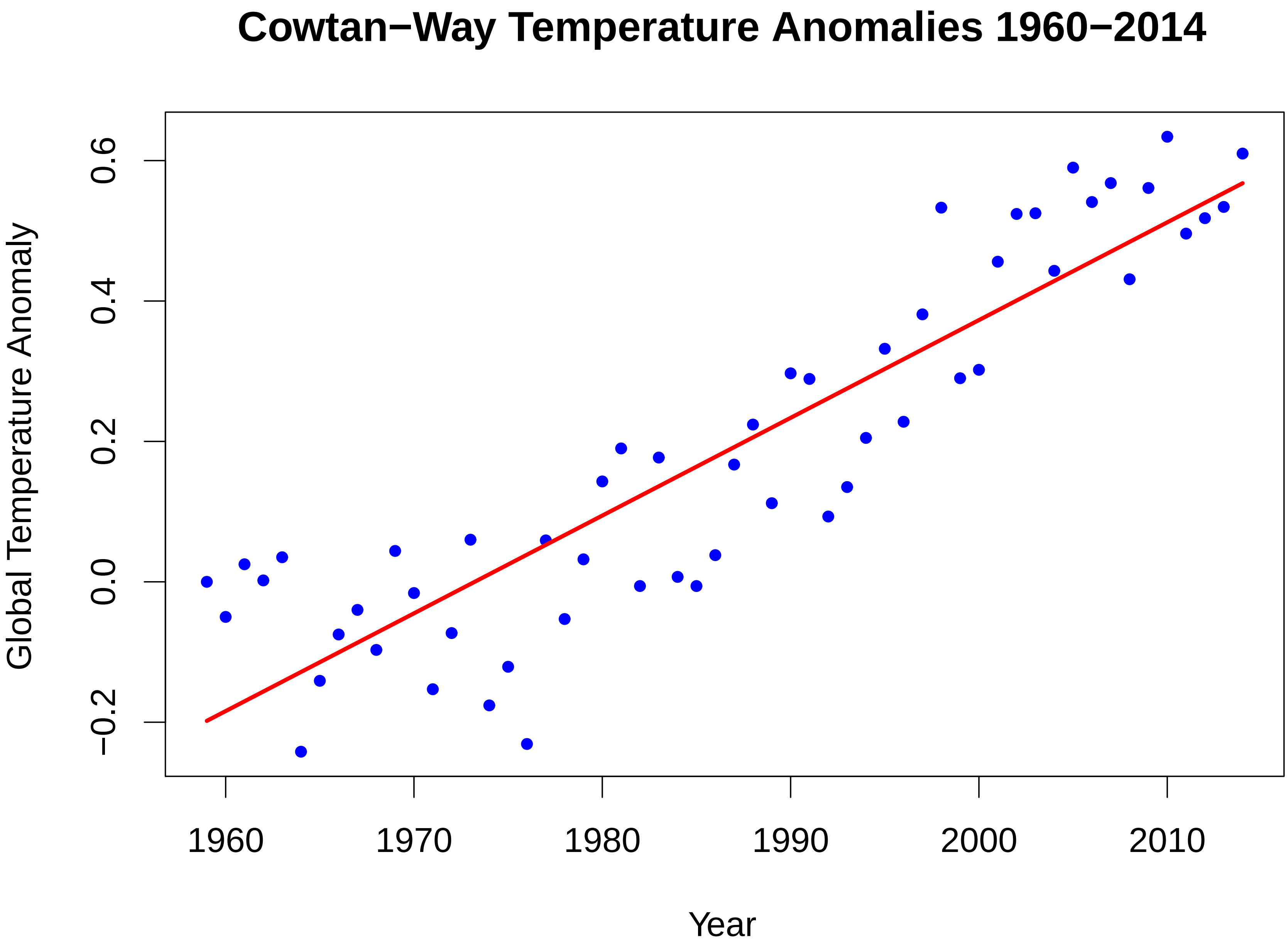

NOAA Temperature Anomalies 1960−2014
Global Temperature Anomaly
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year

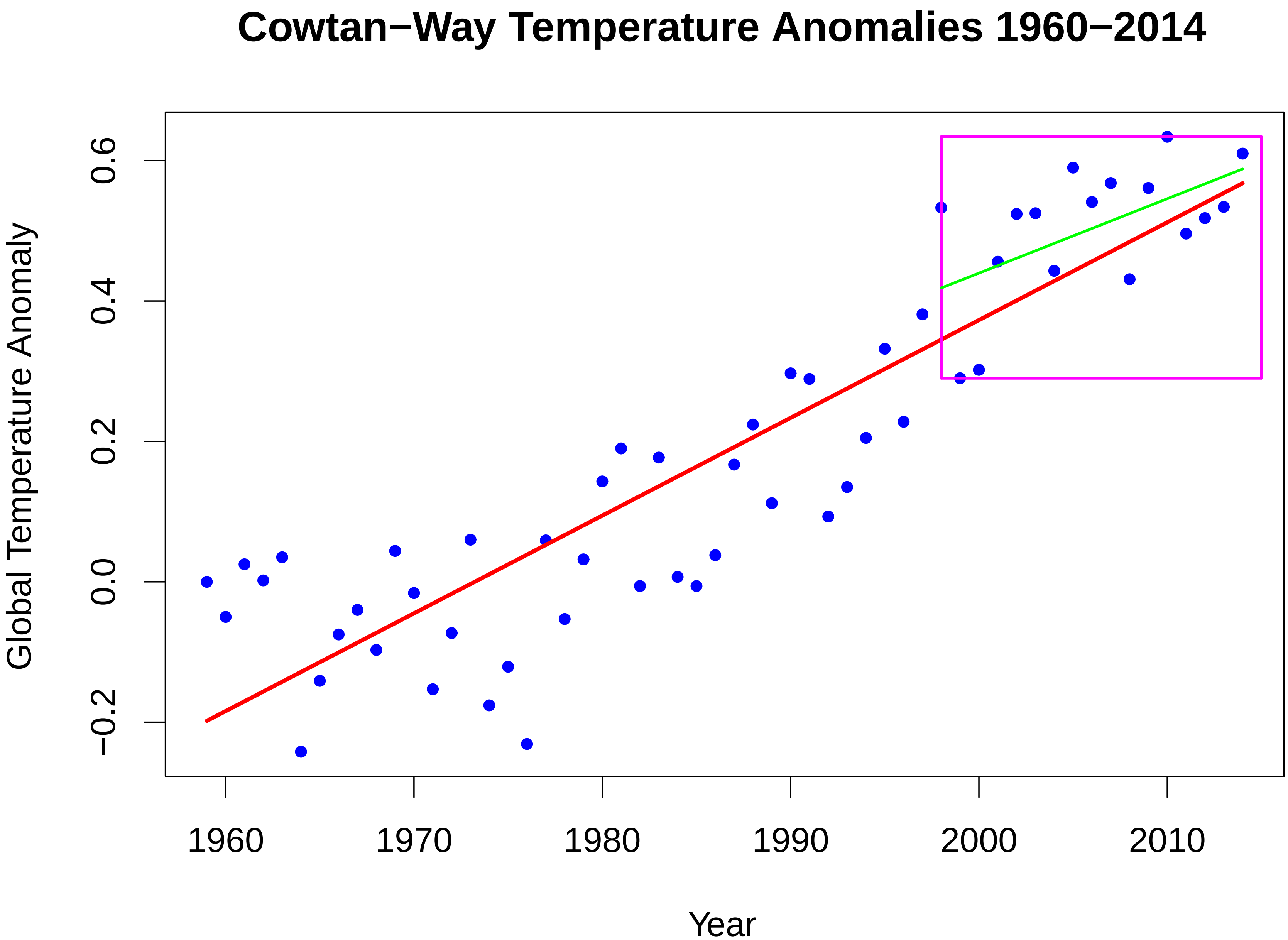

GISS (NASA) Temperature Anomalies 1960−2014
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year


GISS (NASA) Temperature Anomalies 1960−2014
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year


Berkeley Earth Temperature Anomalies 1960−2014
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year


Berkeley Earth Temperature Anomalies 1960−2014
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year


Cowtan−Way Temperature Anomalies 1960−2014
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year

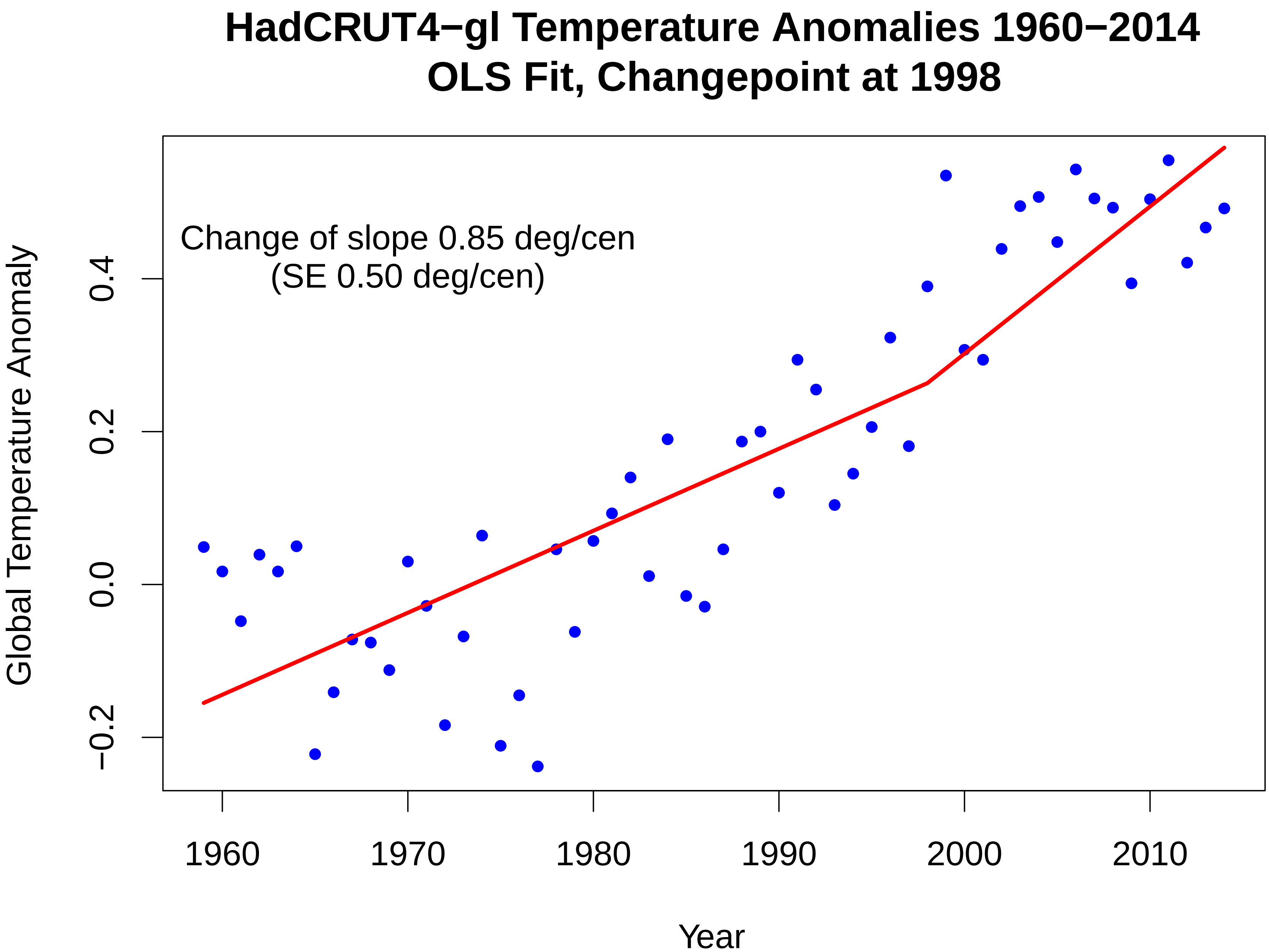

Cowtan−Way Temperature Anomalies 1960−2014
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year

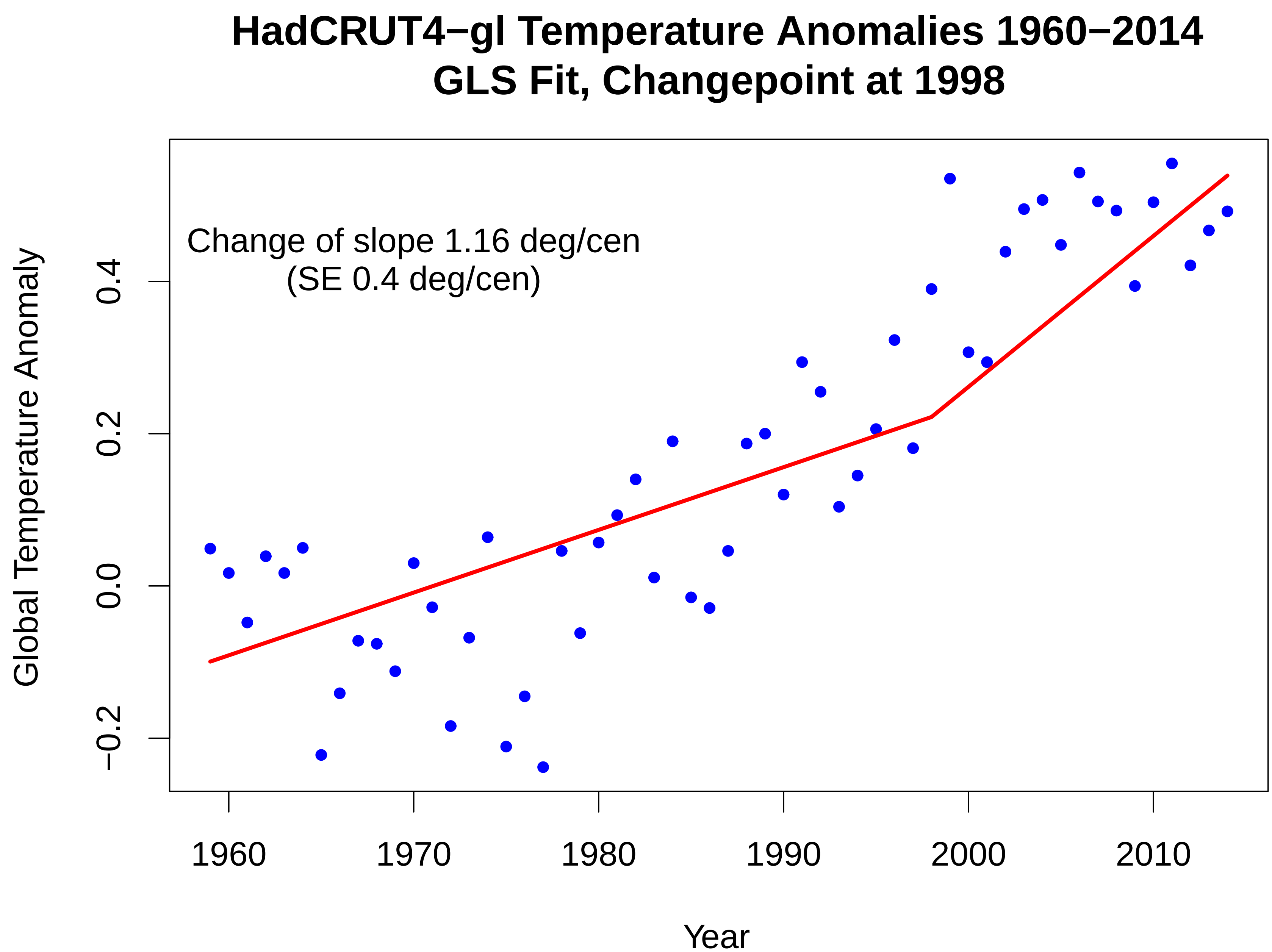
Statistical Models

Let

- t_{1i}: ith year of series
- y_i: temperature anomaly in year t_i
- $t_{2i} = (t_{1i} - 1998)_+$
- $y_i \; = \; \beta_0 + \beta_1 t_{1i} + \beta_2 t_{2i} + u_i$
- Simple linear regression (OLS): $u_i \sim N[0, \sigma^2]$ (IID)
- Time series regression (GLS): $u_i - \phi_1 u_{i-1} - ... - \phi_p u_{i-p} = \epsilon_i + \theta_1 \epsilon_{i-1} + ... + \theta_q \epsilon_{i-q}, \; \epsilon_i \sim N[0, \sigma^2]$ (IID)

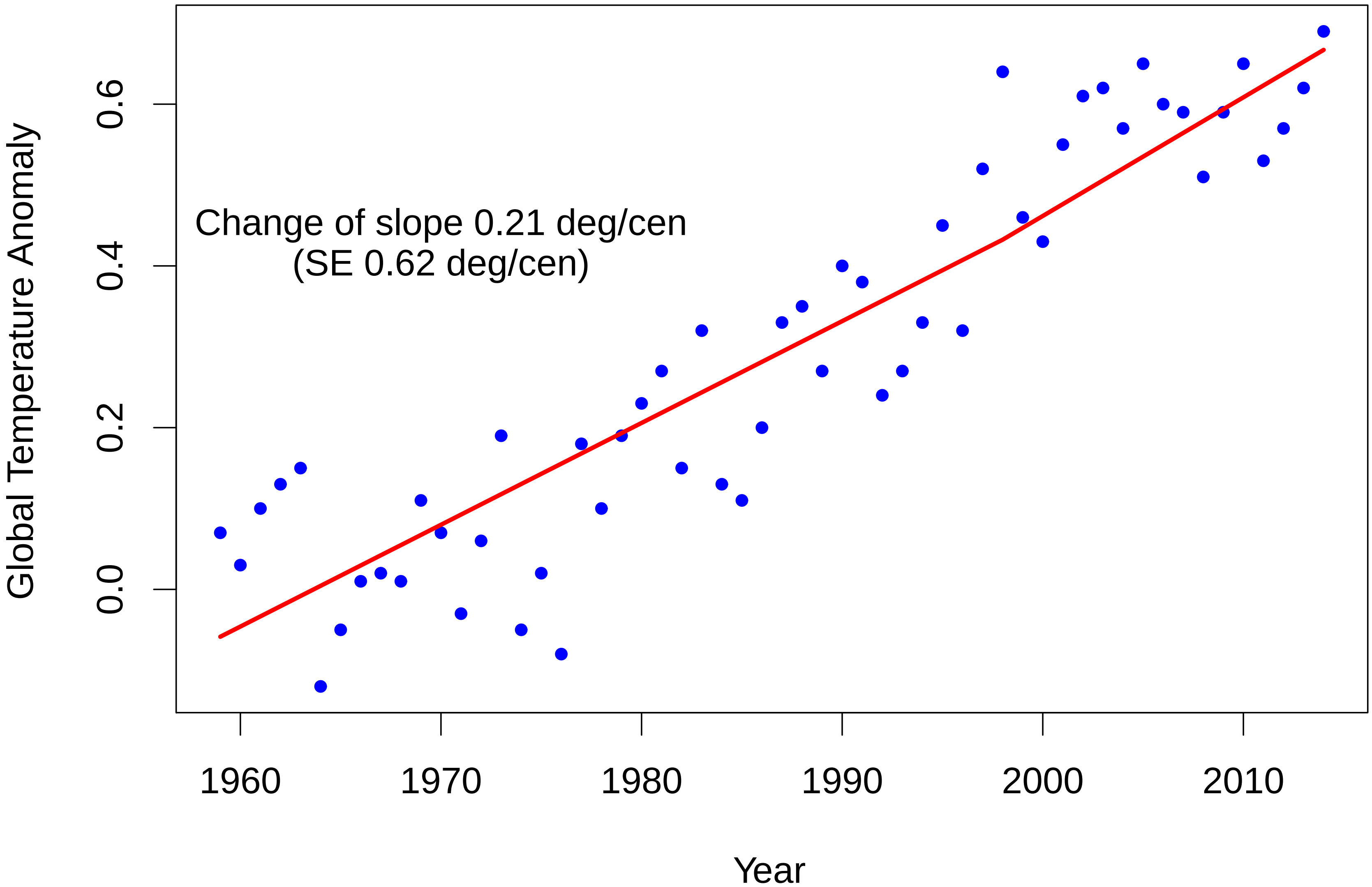
Fit using `arima` function in R


HadCRUT4−gl Temperature Anomalies 1960−2014
OLS Fit, Changepoint at 1998
Change of slope 0.85 deg/cen
(SE 0.50 deg/cen)
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
1960
1970
1980
1990
2000
2010
Year

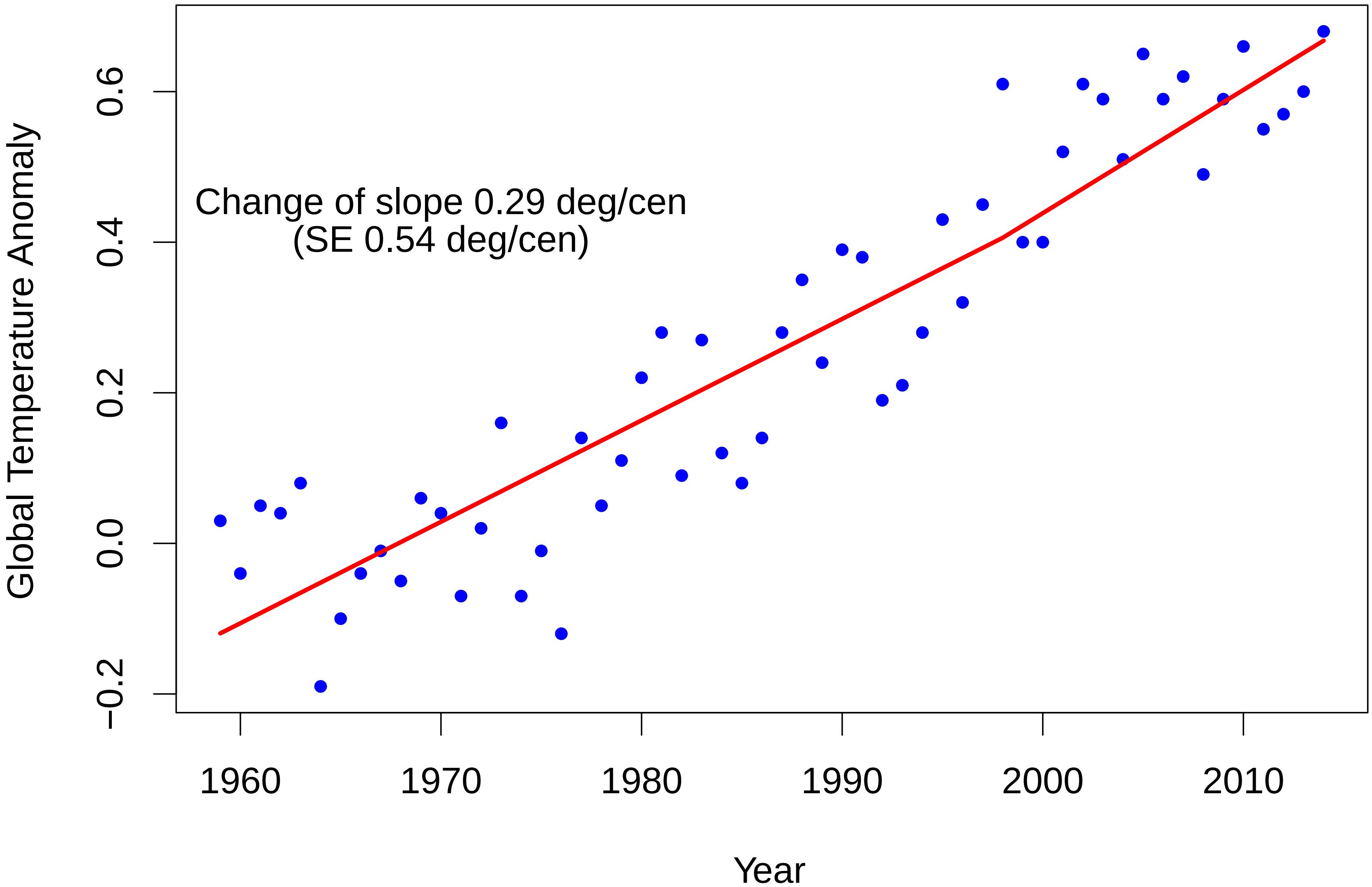


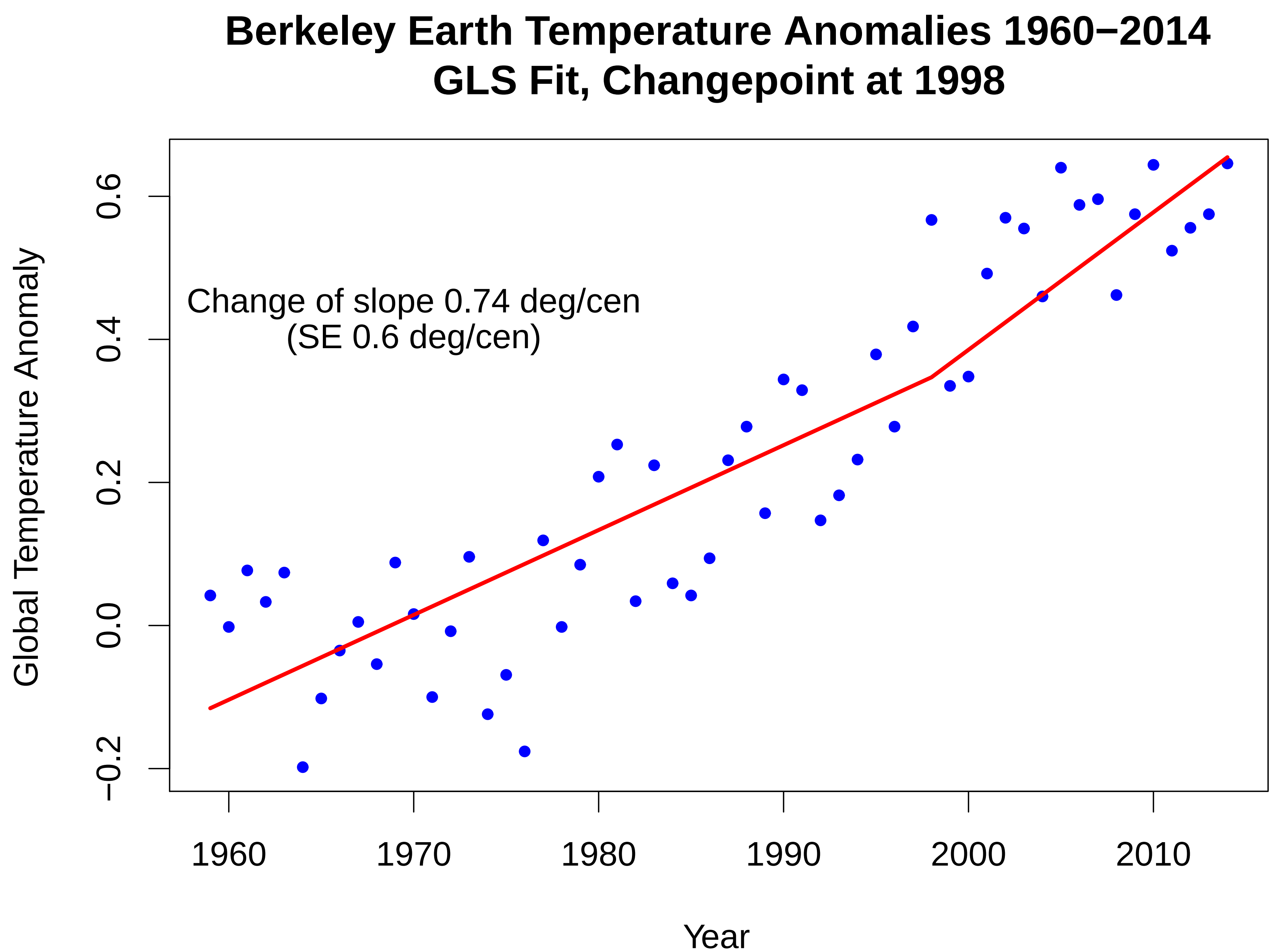
HadCRUT4−gl Temperature Anomalies 1960−2014
GLS Fit, Changepoint at 1998
Change of slope 1.16 deg/cen
(SE 0.4 deg/cen)
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
1960
1970
1980
1990
2000
2010
Year

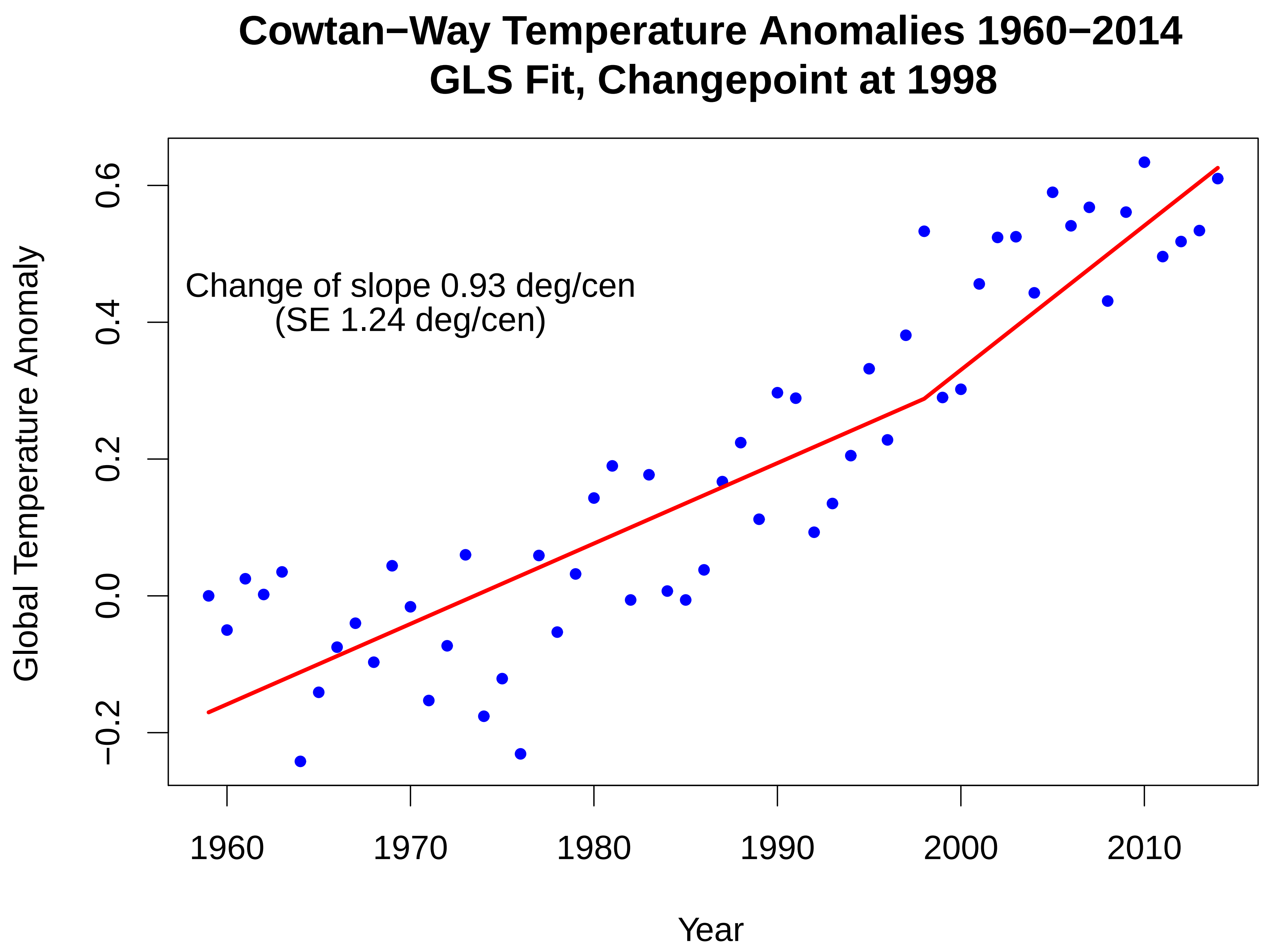
NOAA Temperature Anomalies 1960−2014
GLS Fit, Changepoint at 1998



GISS (NASA) Temperature Anomalies 1960−2014
GLS Fit, Changepoint at 1998

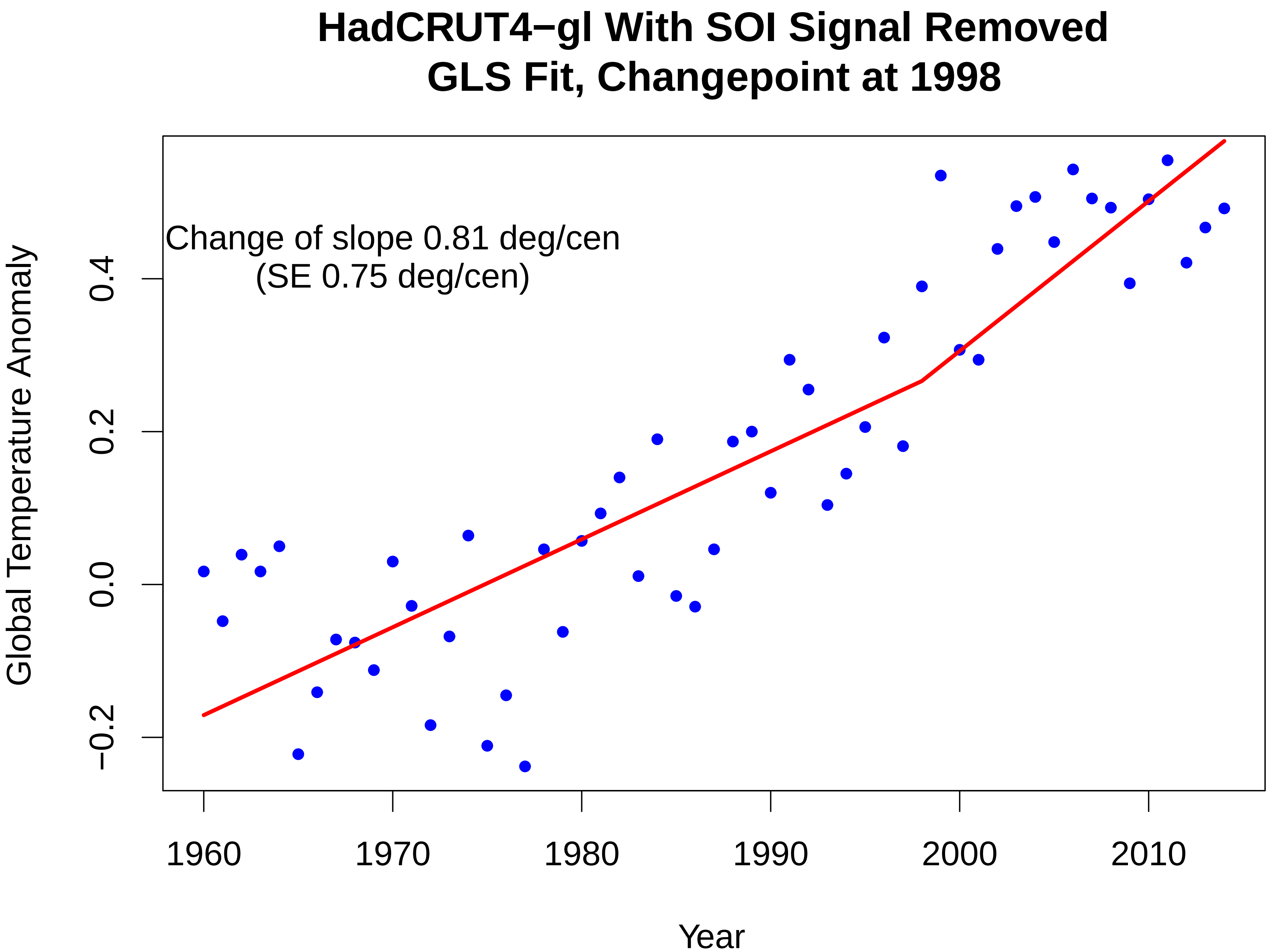


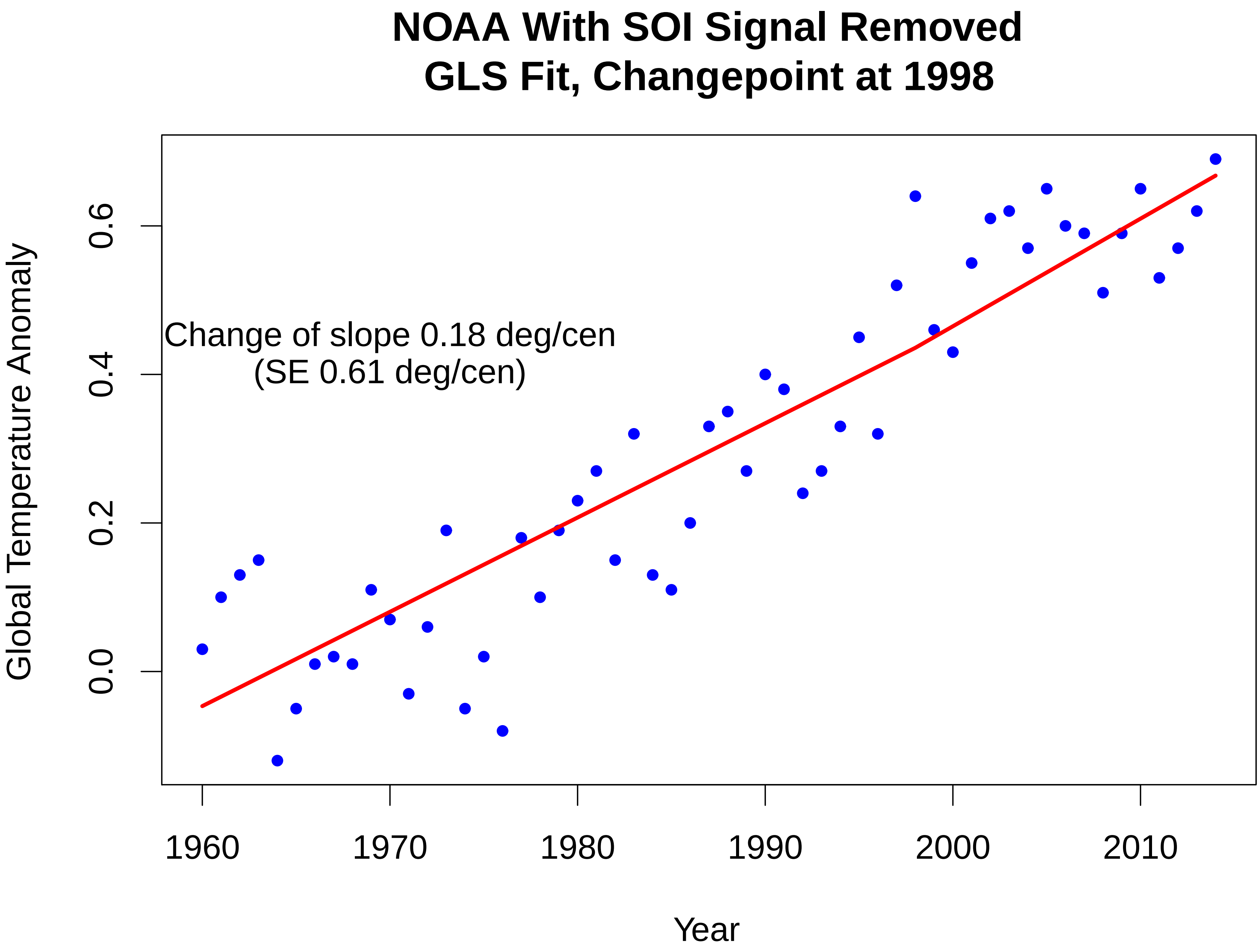

Berkeley Earth Temperature Anomalies 1960−2014
GLS Fit, Changepoint at 1998
Change of slope 0.74 deg/cen
(SE 0.6 deg/cen)
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year


Cowtan−Way Temperature Anomalies 1960−2014
GLS Fit, Changepoint at 1998
Change of slope 0.93 deg/cen
(SE 1.24 deg/cen)
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year

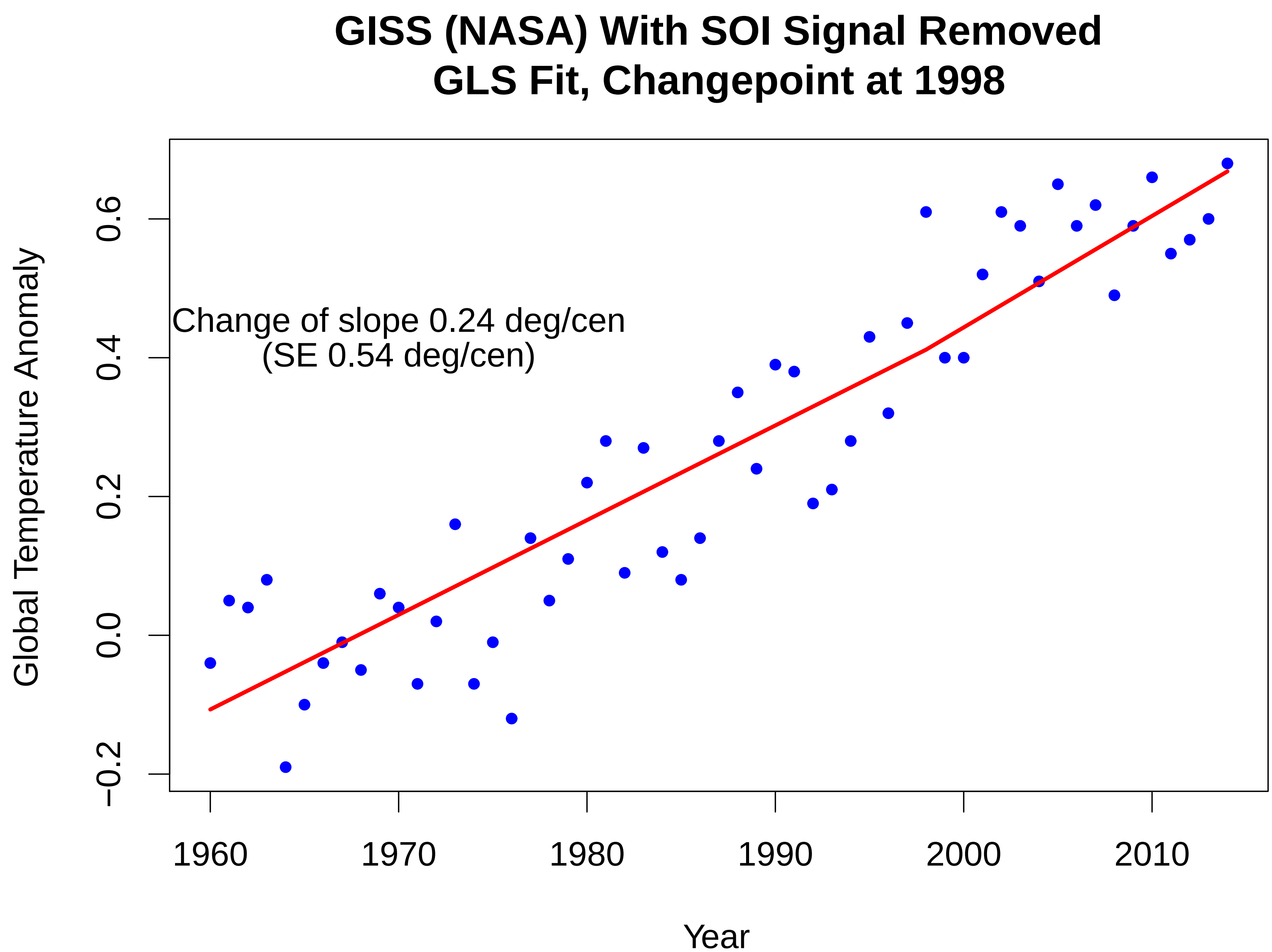
Adjustment for the El Niño Effect

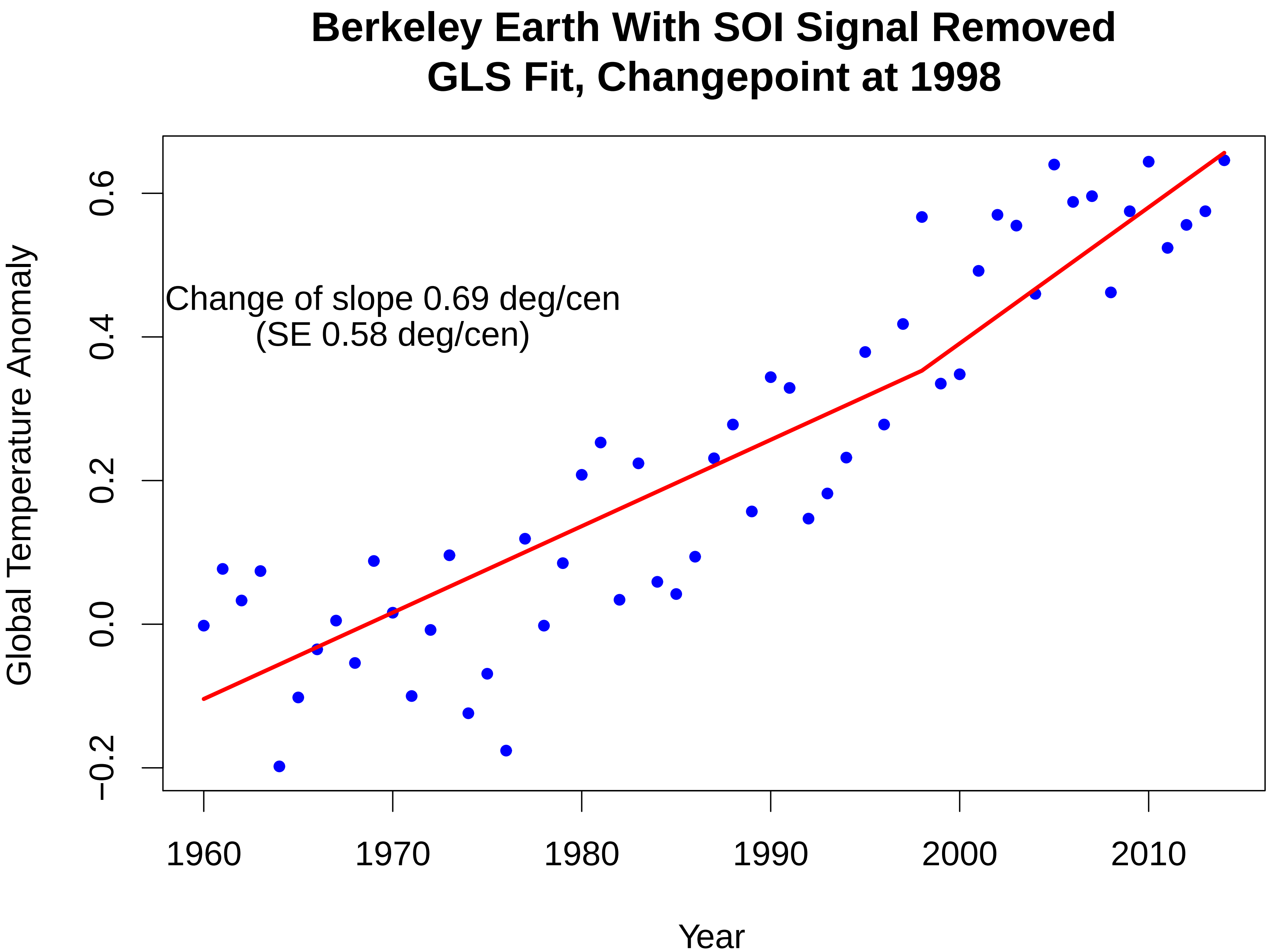
- El Niño is a weather effect caused by circulation changes in the Pacific Ocean
- 1998 was one of the strongest El Niño years in history
- A common measure of El Niño is the *Southern Oscillation Index* (SOI), computed monthly
- Here use SOI with a seven-month lag as an additional co-variate in the analysis

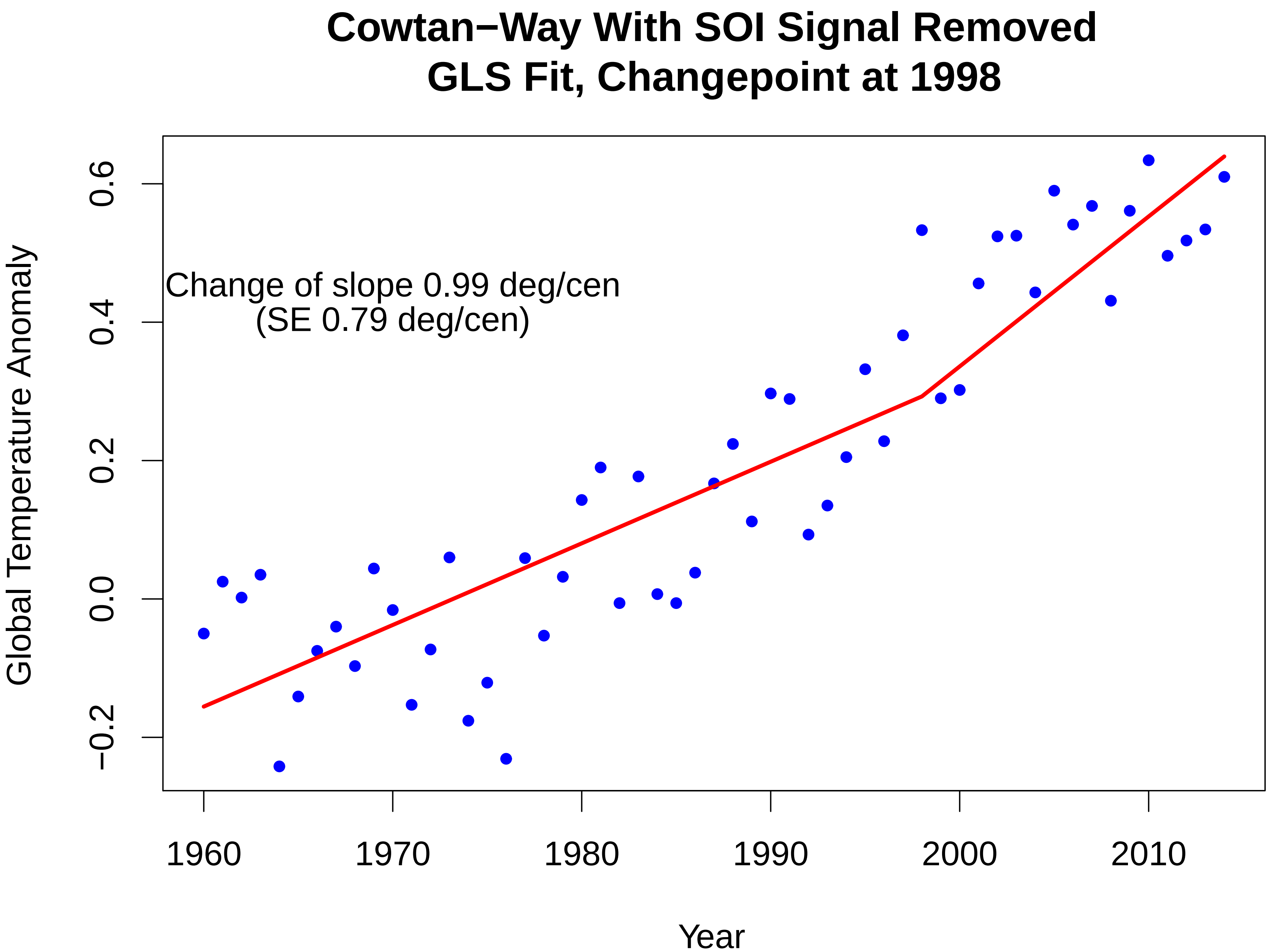

HadCRUT4−gl With SOI Signal Removed
GLS Fit, Changepoint at 1998
Change of slope 0.81 deg/cen
(SE 0.75 deg/cen)
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
1960
1970
1980
1990
2000
2010
Year



NOAA With SOI Signal Removed
GLS Fit, Changepoint at 1998
Change of slope 0.18 deg/cen
(SE 0.61 deg/cen)
Global Temperature Anomaly
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year

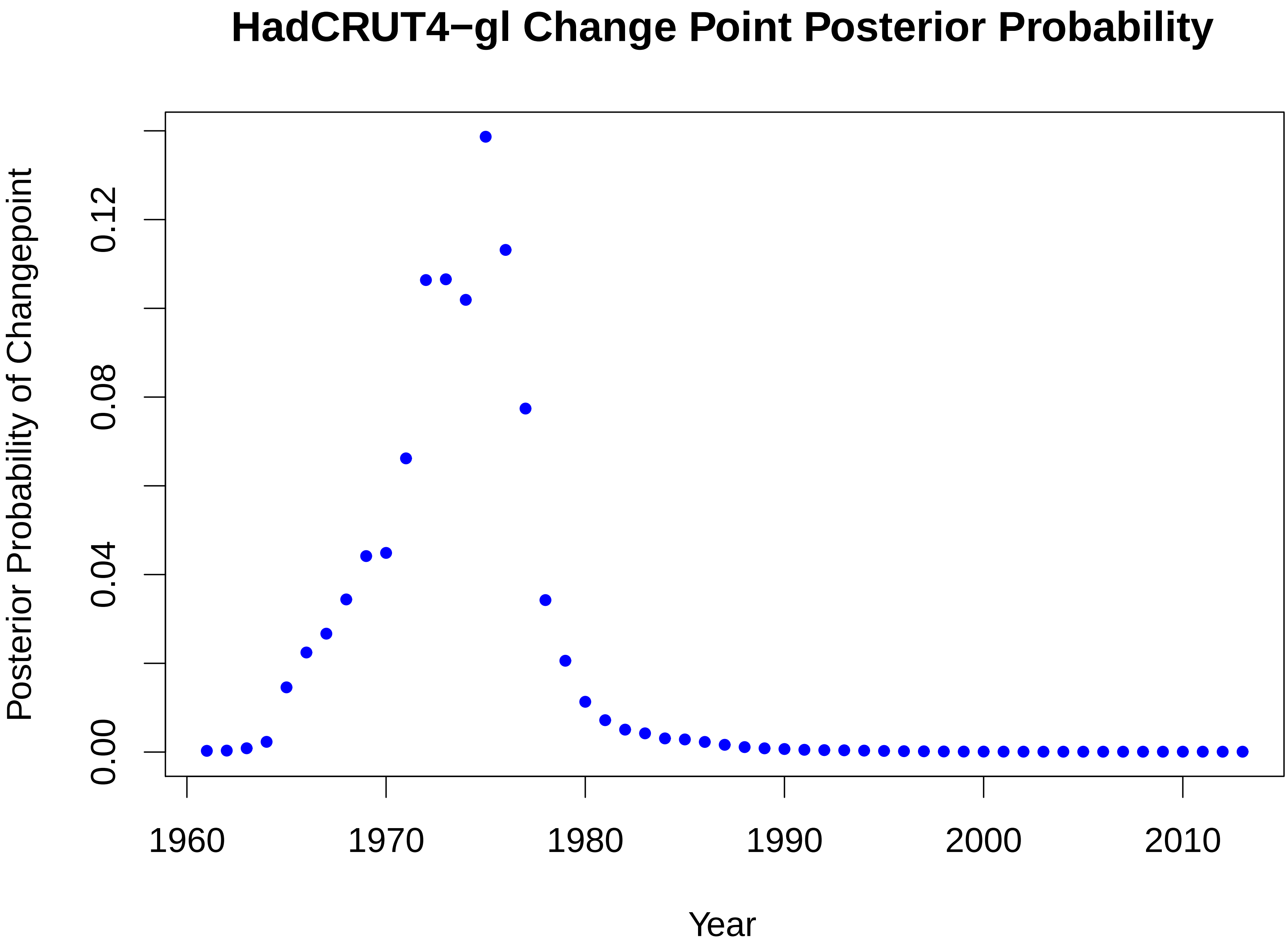

GISS (NASA) With SOI Signal Removed
GLS Fit, Changepoint at 1998
Change of slope 0.24 deg/cen
(SE 0.54 deg/cen)
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year


Berkeley Earth With SOI Signal Removed
GLS Fit, Changepoint at 1998
Change of slope 0.69 deg/cen
(SE 0.58 deg/cen)
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year


Cowtan−Way With SOI Signal Removed
GLS Fit, Changepoint at 1998
Change of slope 0.99 deg/cen
(SE 0.79 deg/cen)
Global Temperature Anomaly
−0.2
0.0
0.2
0.4
0.6
1960
1970
1980
1990
2000
2010
Year

Selecting The Changepoint

If we were to *select* the changepoint through some form of automated statistical changepoint analysis, where would we put it?


HadCRUT4−gl Change Point Posterior Probability
Posterior Probability of Changepoint
0.00
0.04
0.08
0.12
1960
1970
1980
1990
2000
2010
Year

Conclusion from Temperature Trend Analysis

- No evidence of decrease post-1998 — if anything, the trend increases after this time

- After adjusting for El Niño, even stronger evidence for a continuously increasing trend

- If we were to select the changepoint instead of fixing it at 1998, we would choose some year in the 1970s

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AGU PUBLICATIONS

JGR

Journal of Geophysical Research: Atmospheres

RESEARCH ARTICLE
10.1002/2013JD021446

Warm streaks in the U.S. temperature record: What are the chances?

Peter F. Craigmile[1,2], Peter Guttorp[3,4], Robert Lund[5], Richard L. Smith[6,7], Peter W. Thorne[8,9,10], and Derek Arndt[8]

[1]Department of Statistics, Ohio State University, Columbus, Ohio, USA, [2]School of Mathematics and Statistics, University of Glasgow, Scotland, UK, [3]Department of Statistics, University of Washington, Seattle, Washington, USA, [4]Norwegian Computing Center, Oslo, Norway, [5]Department of Mathematical Sciences, Clemson University, Clemson, South Carolina, USA, [6]Department of Statistics and Operations Research, University of North Carolina at Chapel Hill, Chapel Hill, North Carolina, USA, [7]Statistical and Applied Mathematical Sciences Institute, Research Triangle Park, North Carolina, USA, [8]National Climatic Data Center, Asheville, North Carolina, USA, [9]Department of Marine, Earth and Atmospheric Sciences, North Carolina State University at Raleigh, Raleigh, North Carolina, USA, [10]Nansen Environment and Remote Sensing Center, Bergen, Norway

Key Points:

- Estimating the probability of upper tercile events in the U.S. temperature record
- The probability of the event depends on the statistical model fitted
- We consider statistical models with trend, seasonality, and serial dependence

Supporting Information:

- Readme
- Figures S1–S7, Tables S1–S8, and Text S1
- U.S. CONUS temperatures
- Code S1
- Code S2
- Code S3
- Code S4
- Code S5
- Code S6
- Code S7
- Code S8
- Code S9
- Code S10
- Code S11
- Data S1

Abstract A recent observation in NOAA's National Climatic Data Center's monthly assessment of the state of the climate was that contiguous U.S. average monthly temperatures were in the top third of monthly ranked historical temperatures for 13 straight months from June 2011 to June 2012. The chance of such a streak occurring randomly was quoted as $(1/3)^{13}$, or about one in 1.6 million. The streak continued for three more months before the October 2012 value dropped below the upper tercile. The climate system displays a degree of persistence that increases this probability relative to the assumption of independence. This paper puts forth different statistical techniques that more accurately quantify the probability of this and other such streaks. We consider how much more likely streaks are when an underlying warming trend is accounted for in the record, the chance of streaks occurring anywhere in the record, and the distribution of the record's longest streak.

Continental US monthly temperatures, Jan 1895–Oct 2012.

For each month between June 2011 and Sep 2012, the monthly temperature was in the top tercile of all observations for that month up to that point in the time series. Attention was first drawn to this in June 2012, at which point the series of top tercile events was 13 months long, leading to a naïve calculation that the probability of that event was $(1/3)^{13} = 6.3 \times 10^{-7}$. Eventually, the streak extended to 16 months, but ended at that point, as the temperature for Oct 2012 was not in the top tercile.

In this study, we estimate the probability of either a 13-month or a 16-month streak of top-tercile events, under various assumptions about the monthly temperature time series.

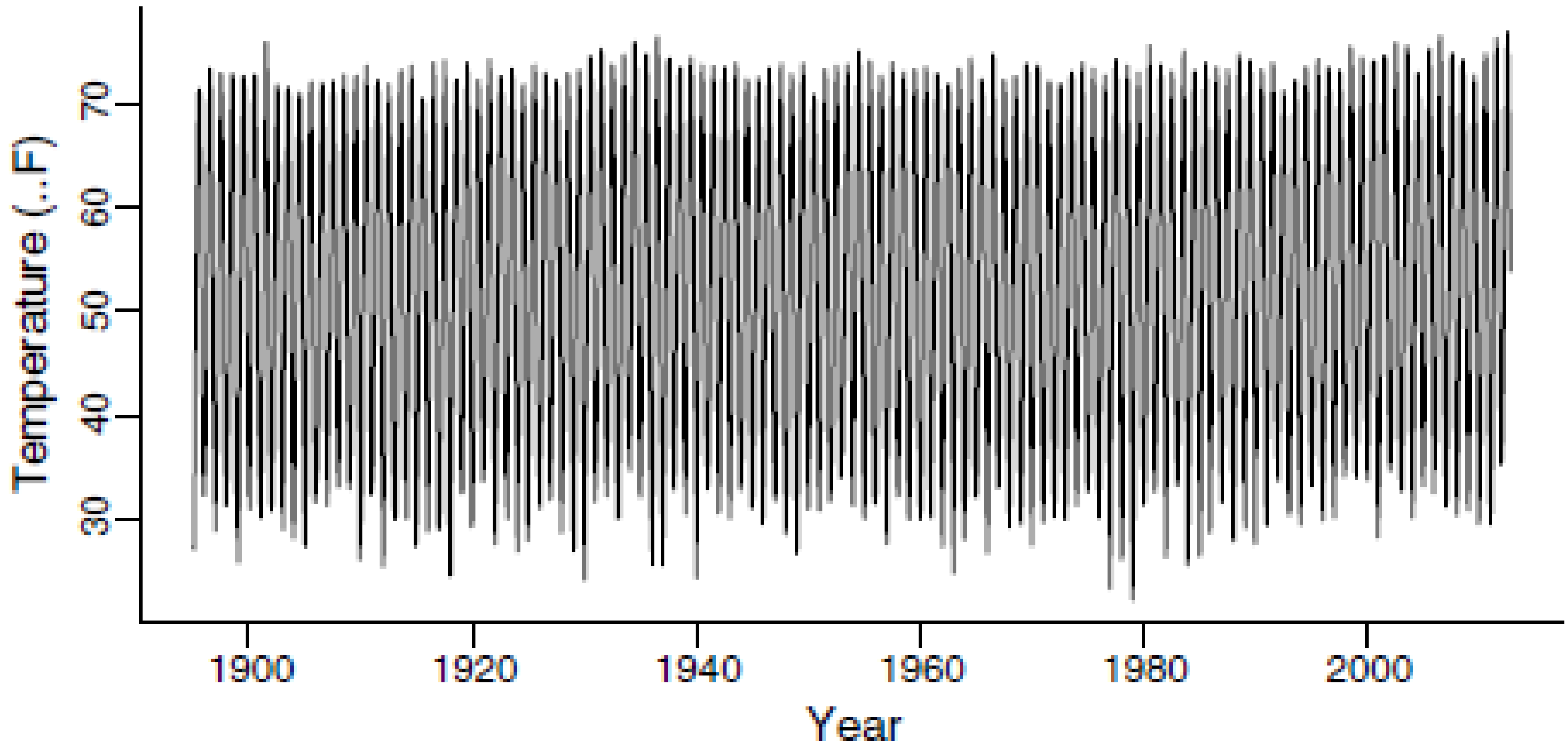

Figure 1. Time series of CONUS average monthly temperatures used in undertaking regular monthly NCDC monitoring reports. Version used: November 2012 report.

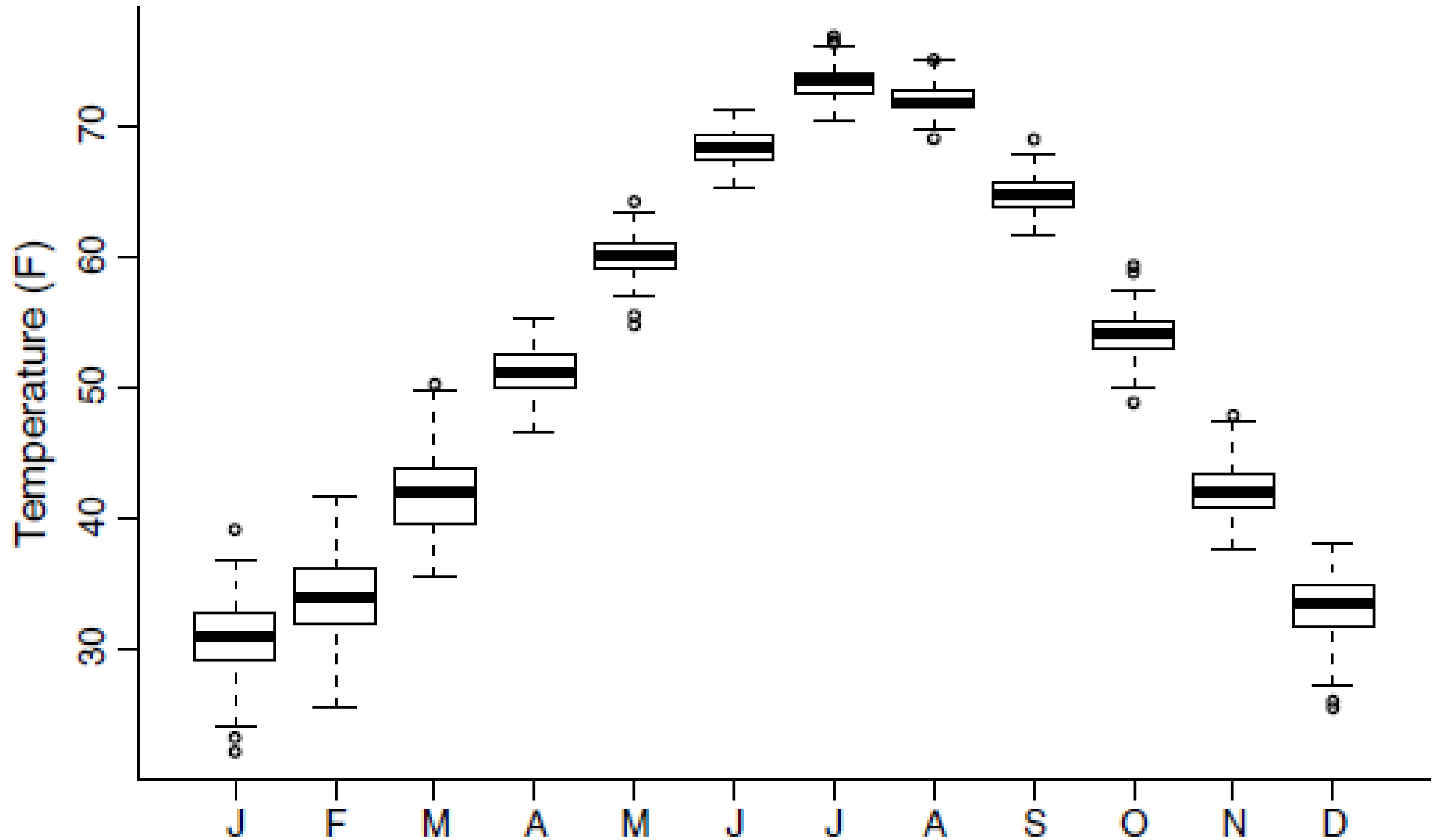

Figure 2. Side-by-side box plots [*Tukey*, 1977] of CONUS average monthly temperatures in °F. The thick horizontal line is the median, the box indicates the first and third quartiles (Q_1 and Q_3), and the whisker extends to the most extreme data point within 1.5 box heights (1.5 times the interquartile range, Q_3–Q_1). Remaining (even more extreme) data are plotted as circles.

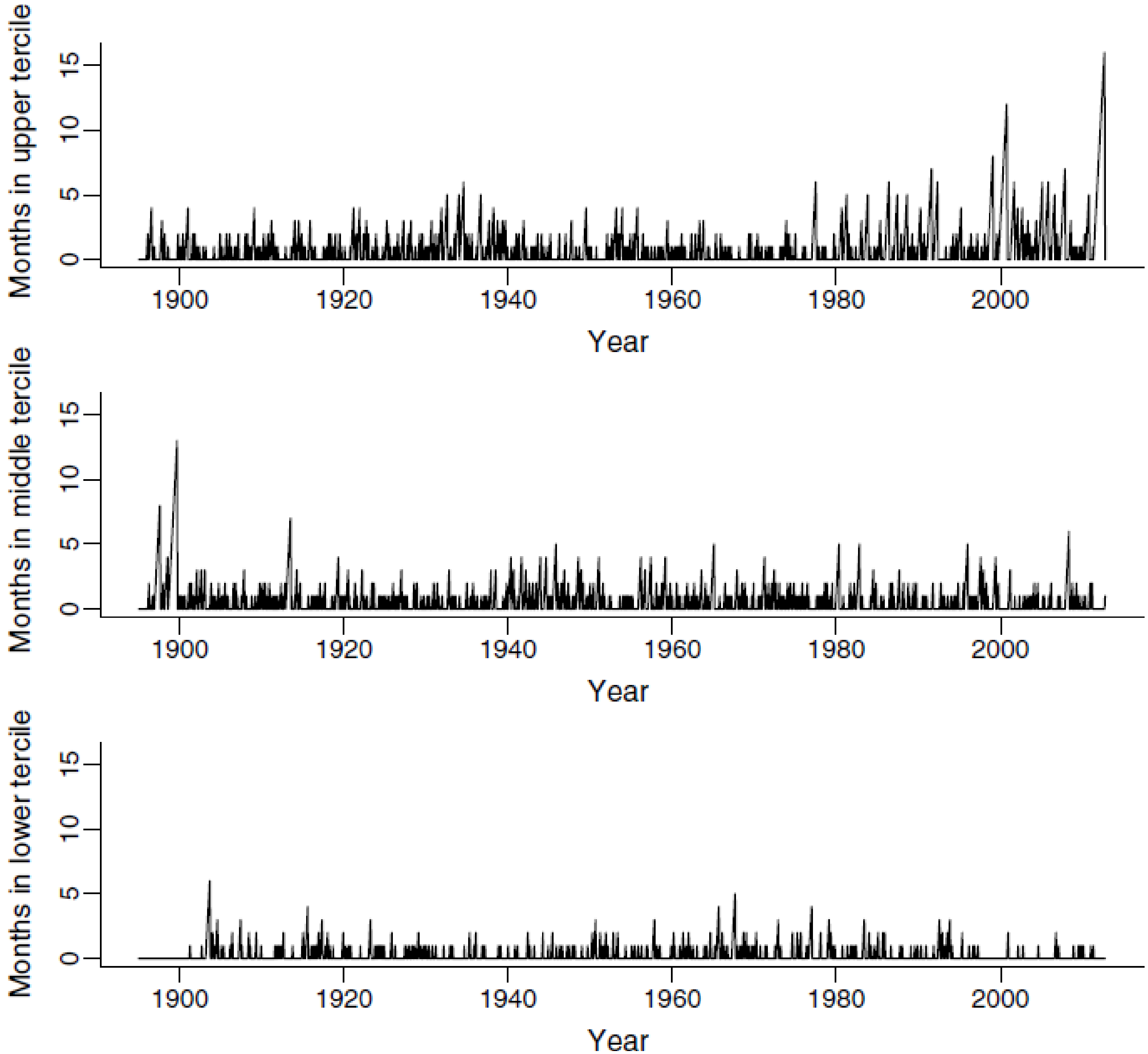

Figure 3. Time series plots of the number of consecutive months in the lower, middle, and upper terciles for the CONUS average monthly temperature record.

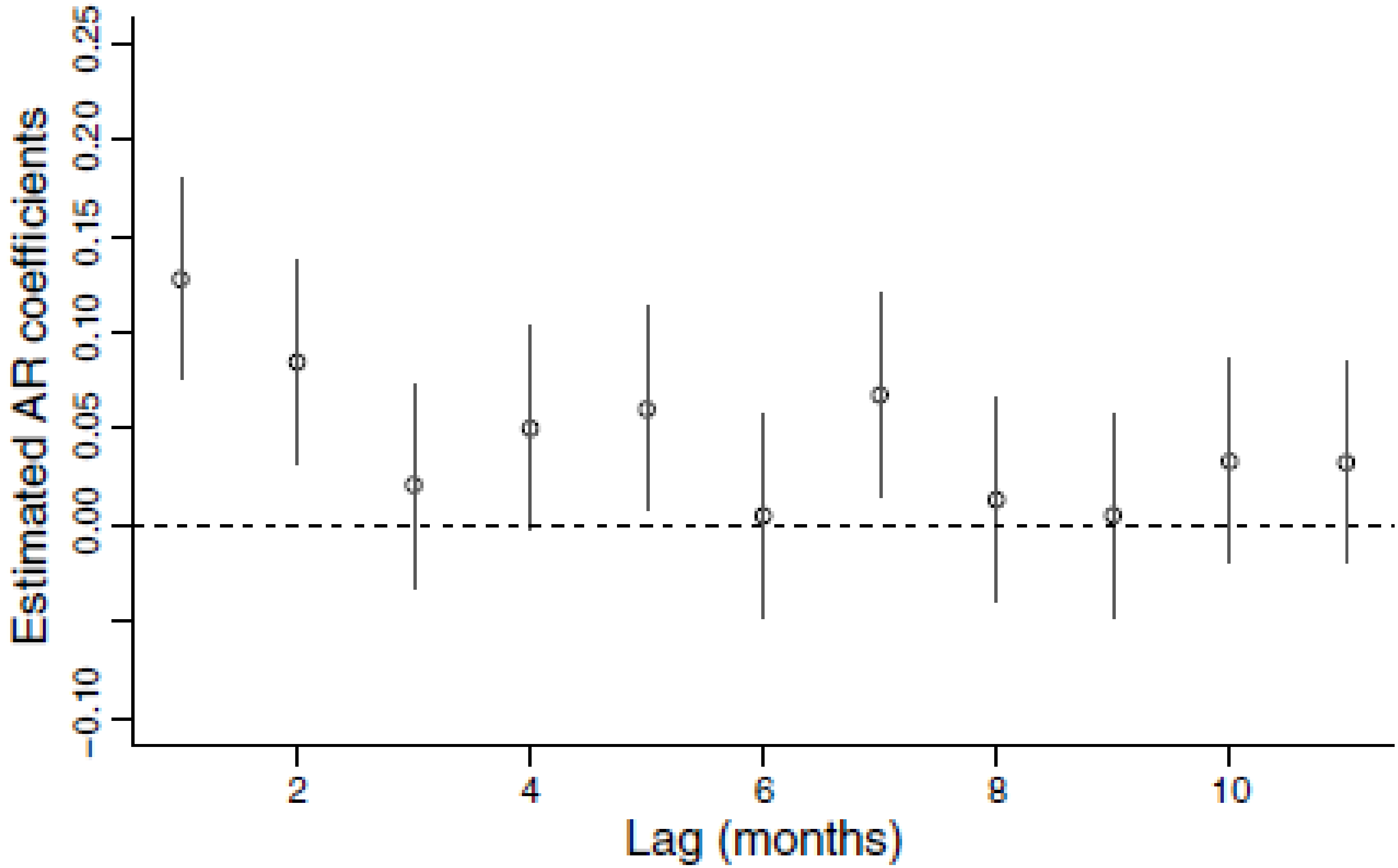

Figure 4. Estimated autoregressive (AR) coefficients for the monthly mean CONUS corrected series. The vertical lines are asymptotic pointwise 95% confidence intervals using the mle option in the ar function in R [*R Core Team*, 2012].

Method

- Two issues with NOAA analysis:
  - Neglects autocorrelation
  - Ignores selection effect

- Solutions:
  - Fit time series model – ARMA or long-range dependence
  - Use simlulation to determine the probability distribution of the longest streak in 117 years

- Some of the issues:
  - Selection of ARMA model — AR(1) performs poorly
  - Variances differ by month — must take that into account
  - Choices of estimation methods, e.g. MLE 0r Bayesian — Bayesian methods allow one to take account of parameter estimation uncertainty

Table 2. Estimate of the Probability of Obtaining an Upper Tercile Streak of at Least 16 Months, Assuming Different Statistical Models for the Temperature[a]

	Assumption for $\{Z_t\}$			
Model	ARMA(3,1)	ARMA(4,2)	FD-ML	FD-Bayes
Stationary model (1)	0.031	0.034	0.019	0.035
Trend model (2)	0.065	0.069	0.116	0.145
Model (2), zero slope	0.007	0.008	0.008	0.013
Nonlinear trend model (3)	0.135	0.141	0.269	0.163
% Increase	830	790	1260	1030

[a]The last line shows the percentage increase in the probability as we go from model (2) with a zero slope to model (2) with the actual slope observed for the temperature series. The first three columns are taken from Table S8 of the supporting information and agree (subject to the margin of error) with results presented in Table S3 and on p. 12 of Text S1. The last column for the fractionally differenced Bayesian model is taken from the $p = 0$ case of Figure S5.

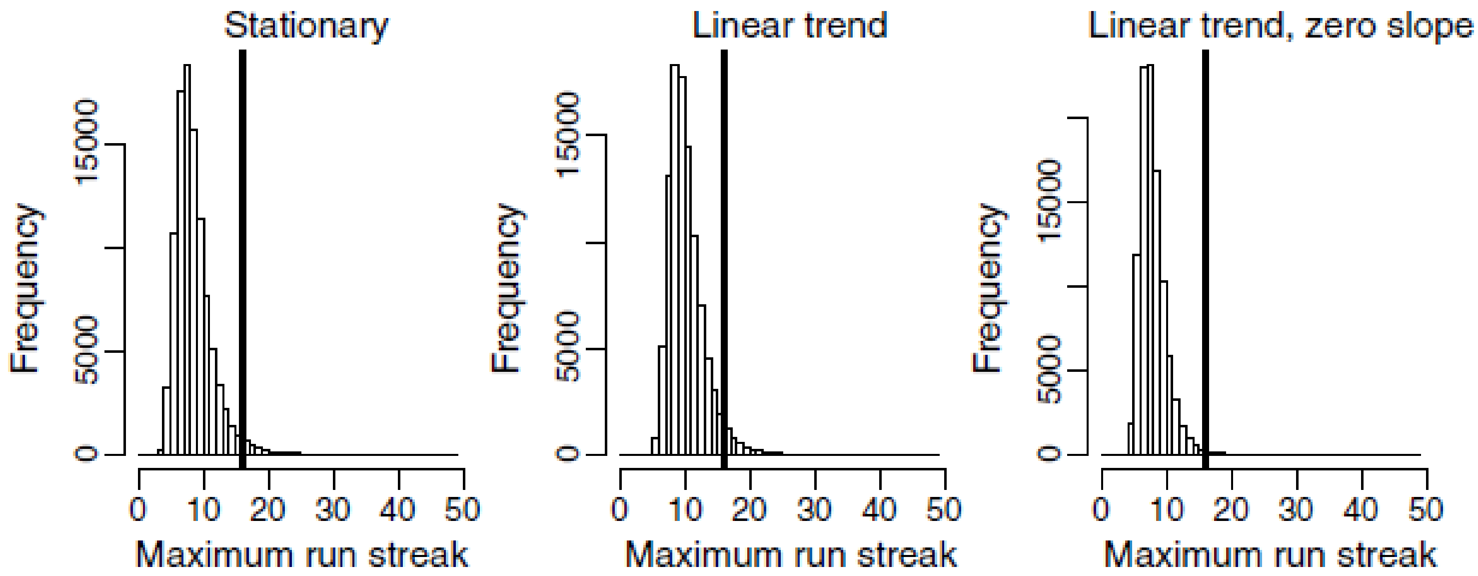

Figure 6. Histograms of the maximum run of upper tercile streak when $\{Z_t\}$ is an ARMA(3,1) process for different assumptions made for the trend.

Conclusions

- It's important to take account of monthly varying standard deviations as well as means.
- Estimation under a high-order ARMA model or fractional differencing lead to very similar results, but don't use AR(1).
- In a model with no trend, the probability that there is a sequence of length 16 consecutive top-tercile observations somewhere after year 30 in the 117-year time series is of the order of 0.01–0.03, depending on the exact model being fitted. With a linear trend, these probability rise to something over .05. Include a *nonlinear* trend, and the probabilities are even higher — in other words, not surprising at all.
- Overall, the results may be taken as supporting the overall anthropogenic influence on temperature, but not to a stronger extent than other methods of analysis.

I. TIME SERIES ANALYSIS FOR CLIMATE DATA

I.a Overview

I.b The post-1998 “hiatus” in temperature trends

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I.d Trends or nonstationarity?

A Parliamentary Question is a device where any member of the U.K. Parliament can ask a question of the Government on any topic, and is entitled to expect a full answer.

Climate Change

Question

Asked by ***Lord Donoughue***

To ask Her Majesty's Government, further to the Written Answers by Baroness Verma on 14 January (*WA 110*), 5 February (*WA 31–2*) and 21 March (*WA 170–1*), whether they will ensure that their assessment of the probability in relation to global temperatures of a linear trend with first-order autoregressive noise compared with a driftless third-order autoregressive integrated model is published in the Official Report; and, if not, why not. [HL6620]

22 Apr 2013 : Column WA359

Lord Newby: As indicated in a previous Written Answer given by my noble friend Baroness Verma to the noble Lord on 14 January 2013 (*Official Report*, col. *WA110*), it is the role of the scientific community to assess and decide between various methods for studying global temperature time series. It is also for the scientific community to publish the findings of such work, in the peer-reviewed scientific literature.

www.parliament.uk, April 22, 2013

Met Office

Statistical models and the global temperature record

May 2013

Professor Julia Slingo,
Met Office Chief Scientist

Essence of the Met Office Response

- Acknowledged that under certain circumstances an ARIMA(3,1,0) without drift can fit the data better than an AR(1) model with drift, as measured by likelihood

- The result depends on the start and finish date of the series

- Provides various reasons why this should not be interpreted as an argument against climate change

- Still, it didn't seem to me (RLS) to settle the issue beyond doubt

There is a tradition of this kind of research going back some time

Global Warming as a Manifestation of a Random Walk

A. H. GORDON

Flinders Institute for Atmospheric and Marine Science, The Flinders University of South Australia, Bedford Park, South Australia

(Manuscript received 17 April 1990, in final form 31 December 1990)

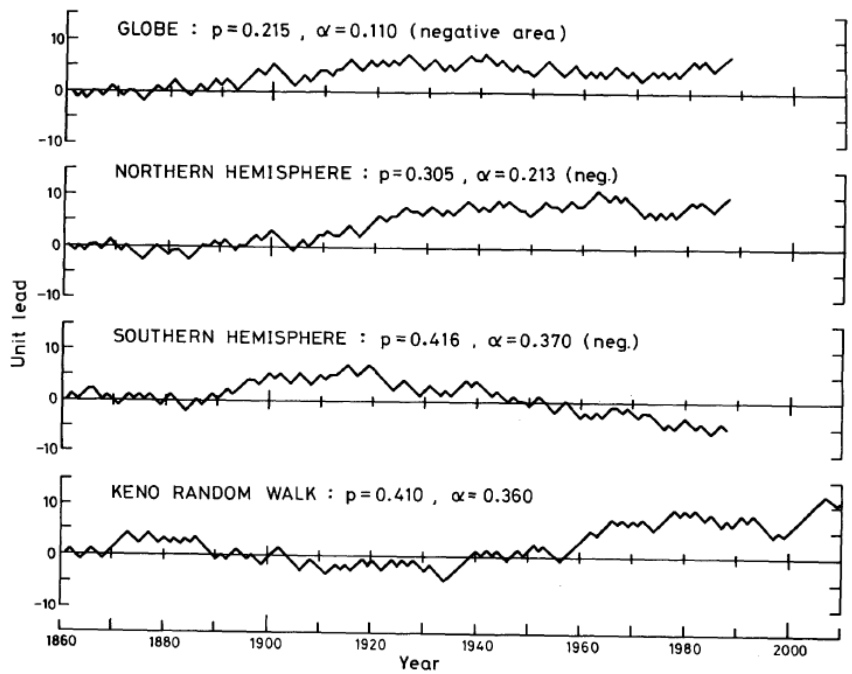


FIG. 3. Plots of the changes in temperature from one year to the next from the 1861–1988 series of mean surface temperature anomalies. Each change has been given unit magnitude. Values of the proportion of time that α spent on the negative y side of the x axis have been calculated from the arc-sine law together with the probabilities associated with those values. The upper plot is from the global series, and the middle two plots are for the two hemispheres, as stated. The probabilities are all within ranges likely to be expected if the plots constituted random walks. The lower plot is a true random walk derived from the sequence of odd and even numbers in a casino gambling game.

It is important to examine all ways and means by which the observed data series develop trends before facing hard and fast conclusions that any particular activity is the one and only responsible agent.

Summary So Far

- Integrated or unit root models (e.g. ARIMA(p,d,q) with $d=1$) have been proposed for climate models and there is some statistical support for them

- If these models are accepted, the evidence for a linear trend is not clear-cut

- Note that we are *not* talking about fractionally integrated models ($0<d<\frac{1}{2}$) for which there is by now a substantial tradition. These models have slowly decaying autocorrelations but are still stationary

- Integrated models are not physically realistic but this has not stopped people advocating them

- I see the need for a more definitive statistical rebuttal

Integrated Time Series Models

HadCRUT4 Global Series, 1900–2012

$$\begin{aligned}
\text{Model I}: \quad y_t - y_{t-1} &= \text{ARMA}(p,q) \text{ (mean 0)} \\
\text{Model II}: \quad y_t &= \text{Linear Trend} + \text{ARMA}(p,q) \\
\text{Model III}: \quad y_t - y_{t-1} &= \text{Nonlinear Trend} + \text{ARMA}(p,q) \\
\text{Model IV}: \quad y_t &= \text{Nonlinear Trend} + \text{ARMA}(p,q)
\end{aligned}$$

Use AICC as measure of fit

Integrated Time Series, No Trend

p	q					
	0	1	2	3	4	5
0	−165.4	−178.9	−182.8	−180.7	−187.7	−185.8
1	−169.2	−181.3	−180.7	−184.1	−186.3	−184.4
2	−176.0	−182.8	−185.7	−182.7	−184.7	−184.4
3	−185.5	−184.2	−185.2	−183.0	−184.4	−184.0
4	−183.5	−181.5	−183.0	−180.7	−181.5	NA
5	−185.2	−183.1	−181.0	−185.8	−183.6	−182.5

Stationary Time Series, Linear Trend

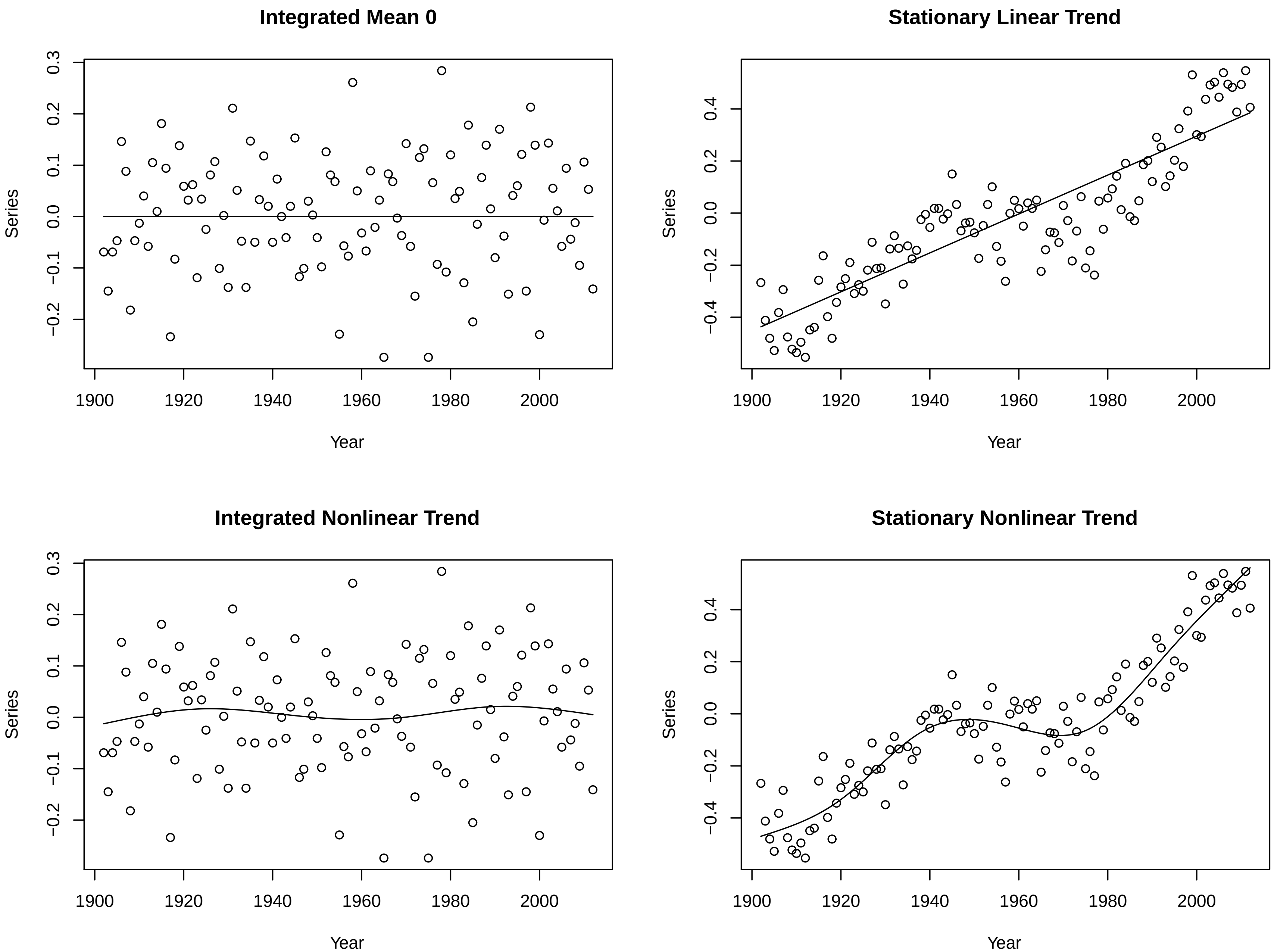
p	q					
	0	1	2	3	4	5
0	−136.1	−168.8	−178.2	−176.2	−180.9	−181.6
1	−183.1	−183.1	−186.8	−184.5	−190.8	−188.5
2	−181.3	−181.7	−184.5	−187.4	−189.2	−187.3
3	−182.6	−186.6	−188.9	−187.1	−189.3	−187.3
4	−189.7	−188.7	−188.4	−185.4	−185.1	NA
5	−187.9	−187.6	−186.0	−183.0	−182.6	−183.8

Integrated Time Series, Nonlinear Trend

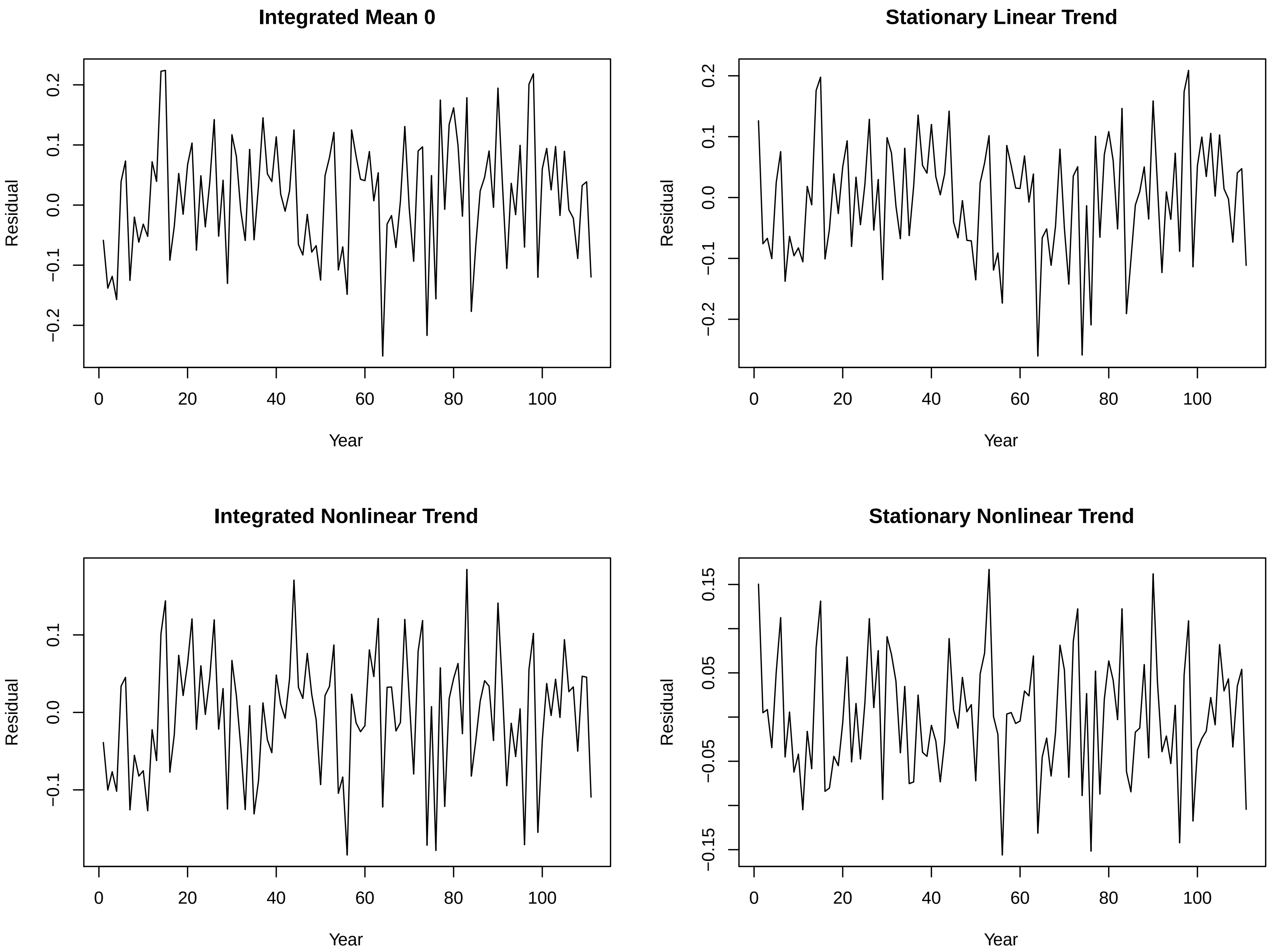
p	q					
	0	1	2	3	4	5
0	−156.8	−195.1	−201.7	−199.4	−207.7	−208.9
1	−161.4	−199.5	−199.4	−202.3	−210.3	−209.0
2	−169.9	−202.3	−210.0	−201.4	−209.7	−208.7
3	−183.2	−201.0	−203.5	−201.2	−207.3	−204.8
4	−180.9	−199.3	−201.2	−198.7	−205.3	NA
5	−186.8	−201.7	−199.4	−207.7	−204.8	−204.8

Stationary Time Series, Nonlinear Trend

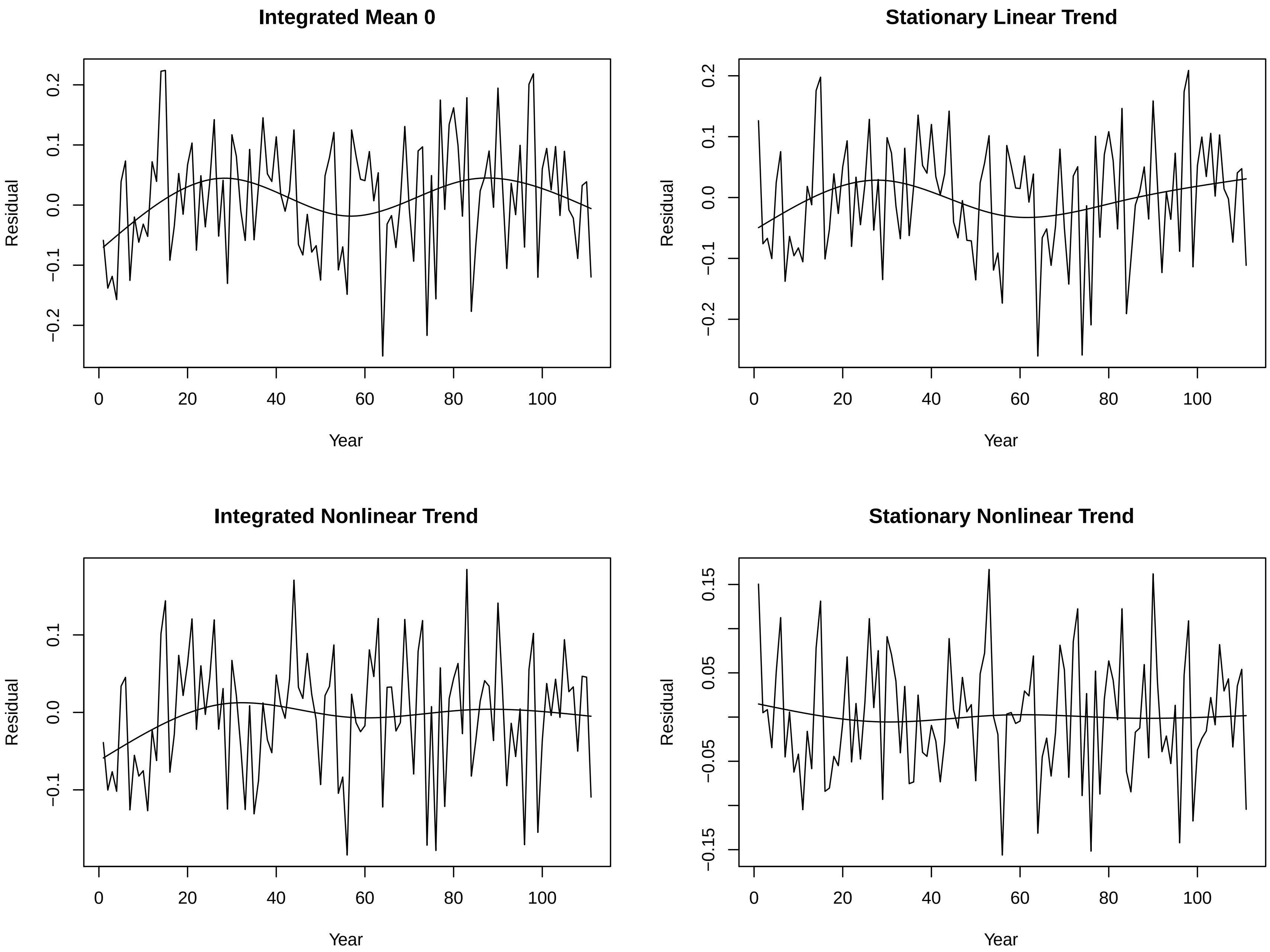
p	q					
	0	1	2	3	4	5
0	−199.1	−204.6	−202.4	−217.8	−216.9	−215.9
1	−202.6	−202.3	−215.2	−217.7	−216.1	−214.7
2	−205.2	−217.3	−205.0	−216.6	−214.1	−213.3
3	−203.8	−205.9	−203.6	−214.3	−211.7	−213.5
4	−202.2	−203.5	−213.7	−212.0	−227.1	NA
5	−205.7	−203.2	−216.2	−233.3	−212.6	−226.5



Four Time Series Models with Fitted Trends



Residuals From Four Time Series Models



Residuals From Four Time Series Models

Conclusions

- If we restrict ourselves to linear trends, there is not a clear-cut preference between integrated time series models without a trend and stationary models with a trend
- However, if we extend the analysis to include nonlinear trends, there is a very clear preference that the residuals are stationary, not integrated
- Possible extensions:
  - Add fractionally integrated models to the comparison
  - Bring in additional covariates, e.g. circulation indices and external forcing factors
  - Consider using a nonlinear trend derived from a climate model. That would make clear the connection with detection and attribution methods which are the preferred tool for attributing climate change used by climatologists.

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I.a Overview

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II. CLIMATE EXTREMES

II.a Extreme value models

EXTREME VALUE DISTRIBUTIONS

$X_1, X_2, ...,$ i.i.d., $F(x) = \Pr\{X_i \le x\}$, $M_n = \max(X_1, ..., X_n)$,

$$\Pr\{M_n \le x\} = F(x)^n.$$

For non-trivial results must *renormalize*: find $a_n > 0, b_n$ such that

$$\Pr\left\{\frac{M_n - b_n}{a_n} \le x\right\} \;=\; F(a_n x + b_n)^n \to H(x).$$

The *Three Types Theorem* (Fisher-Tippett, Gnedenko) asserts that if nondegenerate H exists, it must be one of three types:

$$\begin{aligned} H(x) &= \exp(-e^{-x}), \text{ all } x \quad \text{(Gumbel)} \\ H(x) &= \begin{cases} 0 & x < 0 \\ \exp(-x^{-\alpha}) & x > 0 \end{cases} \text{(Fréchet)} \\ H(x) &= \begin{cases} \exp(-|x|^{\alpha}) & x < 0 \\ 1 & x > 0 \end{cases} \text{(Weibull)} \end{aligned}$$

In Fréchet and Weibull, $\alpha > 0$.

The three types may be combined into a single *generalized extreme value* (GEV) distribution:

$$H(x) = \exp\left\{ - \left(1 + \xi \frac{x-\mu}{\psi} \right)_{+}^{-1/\xi} \right\},$$

$(y_+ = \max(y, 0))$

where μ is a location parameter, $\psi > 0$ is a scale parameter and ξ is a shape parameter. $\xi \to 0$ corresponds to the Gumbel distribution, $\xi > 0$ to the Fréchet distribution with $\alpha = 1/\xi$, $\xi < 0$ to the Weibull distribution with $\alpha = -1/\xi$.

$\xi > 0$: "long-tailed" case, $1 - F(x) \propto x^{-1/\xi}$,

$\xi = 0$: "exponential tail"

$\xi < 0$: "short-tailed" case, finite endpoint at $\mu - \xi/\psi$

EXCEEDANCES OVER THRESHOLDS

Consider the distribution of X conditionally on exceeding some high threshold u:

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)}.$$

As $u \to \omega_F = \sup\{x : F(x) < 1\}$, often find a limit

$$F_u(y) \approx G(y; \sigma_u, \xi)$$

where G is *generalized Pareto distribution* (GPD)

$$G(y; \sigma, \xi) = 1 - \left(1 + \xi \frac{y}{\sigma}\right)_+^{-1/\xi}.$$

Equivalence to three types theorem established by Pickands (1975).

The Generalized Pareto Distribution

$$G(y;\sigma,\xi) = 1 - \left(1 + \xi \frac{y}{\sigma}\right)_+^{-1/\xi}.$$

$\xi > 0$: long-tailed (equivalent to usual Pareto distribution), tail like $x^{-1/\xi}$,

$\xi = 0$: take limit as $\xi \to 0$ to get

$$G(y;\sigma,0) = 1 - \exp\left(-\frac{y}{\sigma}\right),$$

i.e. exponential distribution with mean σ,

$\xi < 0$: finite upper endpoint at $-\sigma/\xi$.

POISSON-GPD MODEL FOR EXCEEDANCES

1. The number, N, of exceedances of the level u in any one year has a Poisson distribution with mean λ,

2. Conditionally on $N \geq 1$, the excess values $Y_1, ..., Y_N$ are IID from the GPD.

Relation to GEV for annual maxima:

Suppose $x > u$. The probability that the annual maximum of the Poisson-GPD process is less than x is

$$
\begin{aligned}
& \Pr\{\max_{1 \le i \le N} Y_i \le x\} = \Pr\{N = 0\} + \sum_{n=1}^{\infty} \Pr\{N = n,\ Y_1 \le x,\ \dots\ Y_n \le x\} \\
= \ & e^{-\lambda} + \sum_{n=1}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} \left\{ 1 - \left(1 + \xi \frac{x-u}{\sigma} \right)^{-1/\xi} \right\}^n \\
= \ & \exp\left\{ -\lambda \left(1 + \xi \frac{x-u}{\sigma} \right)^{-1/\xi} \right\}.
\end{aligned}
$$

This is GEV with $\sigma = \psi + \xi(u - \mu), \lambda = \left(1 + \xi \frac{u-\mu}{\psi}\right)^{-1/\xi}$. Thus the GEV and GPD models are entirely consistent with one another above the GPD threshold, and moreover, shows exactly how the Poisson–GPD parameters σ and λ vary with u.

ALTERNATIVE PROBABILITY MODELS

1. The r largest order statistics model

If $Y_{n,1} \geq Y_{n,2} \geq ... \geq Y_{n,r}$ are r largest order statistics of IID sample of size n, and a_n and b_n are EVT normalizing constants, then

$$\left(\frac{Y_{n,1}-b_n}{a_n},...,\frac{Y_{n,r}-b_n}{a_n}\right)$$

converges in distribution to a limiting random vector $(X_1,...,X_r)$, whose density is

$$\begin{aligned} h(x_1,...,x_r) &= \psi^{-r}\exp\left\{-\left(1+\xi\frac{x_r-\mu}{\psi}\right)^{-1/\xi}\right. \\ &\qquad \left. -\left(1+\frac{1}{\xi}\right)\sum_{j=1}^{r}\log\left(1+\xi\frac{x_j-\mu}{\psi}\right)\right\}. \end{aligned}$$

2. Point process approach (Smith 1989)

Two-dimensional plot of exceedance times and exceedance levels forms a nonhomogeneous Poisson process with

$$\begin{aligned} \Lambda(A) &= (t_2 - t_1)\Psi(y;\mu,\psi,\xi) \\ \Psi(y;\mu,\psi,\xi) &= \left(1+\xi\frac{y-\mu}{\psi}\right)^{-1/\xi} \end{aligned}$$

$(1+\xi(y-\mu)/\psi > 0)$.

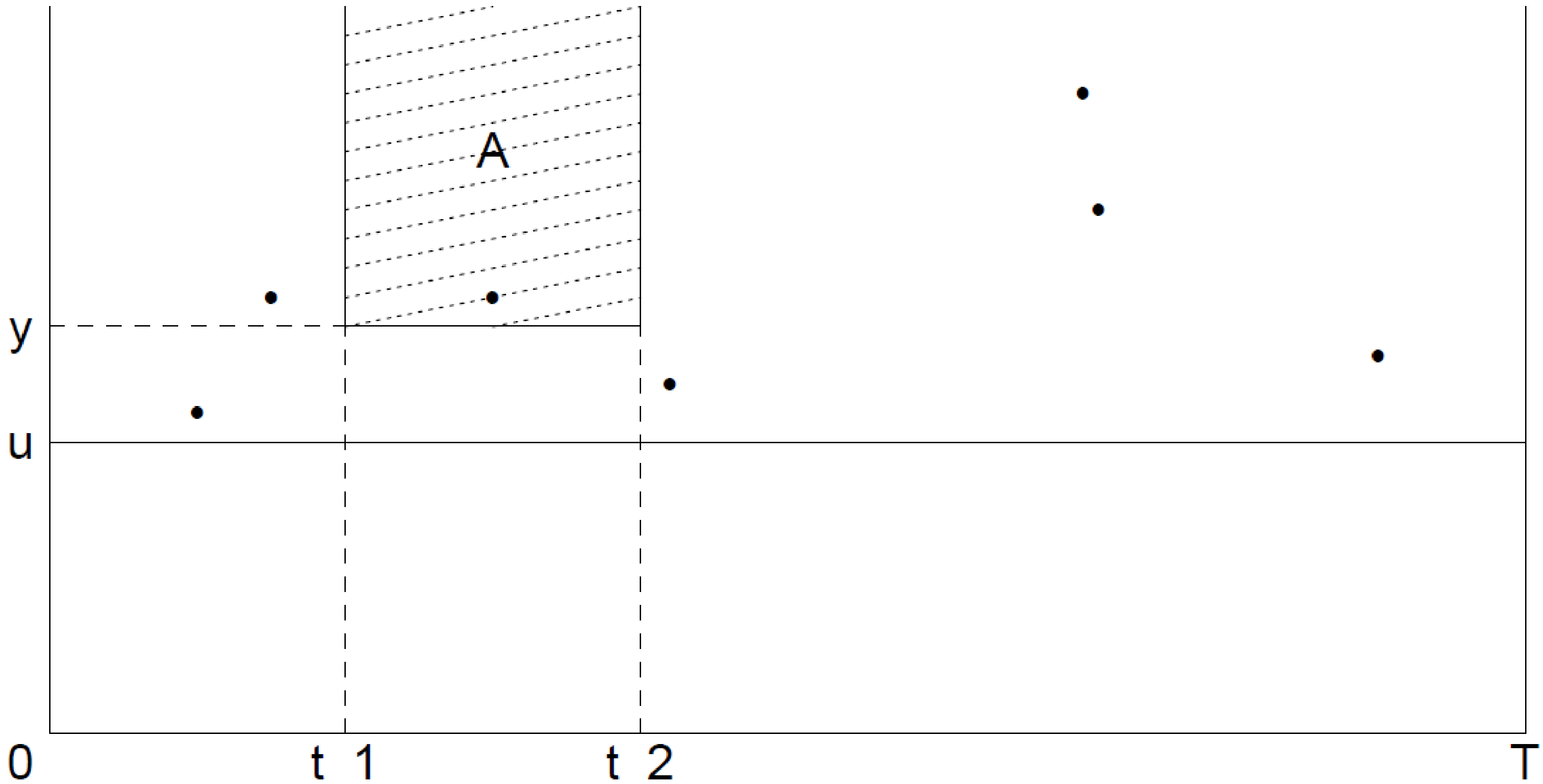

Illustration of point process model.

An extension of this approach allows for nonstationary processes in which the parameters μ, ψ and ξ are all allowed to be time-dependent, denoted μ_t, ψ_t and ξ_t.

This is the basis of the extreme value regression approaches introduced later

Comment. The point process approach is *almost* equivalent to the following: assume the GEV (not GPD) distribution is valid for exceedances over the threshold, and that all observations under the threshold are censored. Compared with the GPD approach, the parameterization directly in terms of μ, ψ, ξ is often easier to interpret, especially when trends are involved.

ESTIMATION

GEV log likelihood:

$$\ell_Y(\mu,\psi,\xi) = -N \log \psi - \left(\frac{1}{\xi}+1\right)\sum_i \log\left(1+\xi\frac{Y_i-\mu}{\psi}\right) - \sum_i \left(1+\xi\frac{Y_i-\mu}{\psi}\right)^{-1/\xi}$$

provided $1+\xi(Y_i-\mu)/\psi > 0$ for each i.

Poisson-GPD model:

$$\ell_{N,Y}(\lambda,\sigma,\xi) = N\log\lambda - \lambda T - N\log\sigma - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{N}\log\left(1+\xi\frac{Y_i}{\sigma}\right)$$

provided $1+\xi Y_i/\sigma > 0$ for all i.

Usual asymptotics valid if $\xi > -\frac{1}{2}$ (Smith 1985)

Bayesian approaches

An alternative approach to extreme value inference is Bayesian, using vague priors for the GEV parameters and MCMC samples for the computations. Bayesian methods are particularly useful for *predictive inference*, e.g. if Z is some as yet unobserved random variable whose distribution depends on μ, ψ and ξ, estimate $\Pr\{Z > z\}$ by

$$\int \Pr\{Z > z; \mu, \psi, \xi\} \pi(\mu, \psi, \xi | Y) d\mu d\psi d\xi$$

where $\pi(...|Y)$ denotes the posterior density given past data Y

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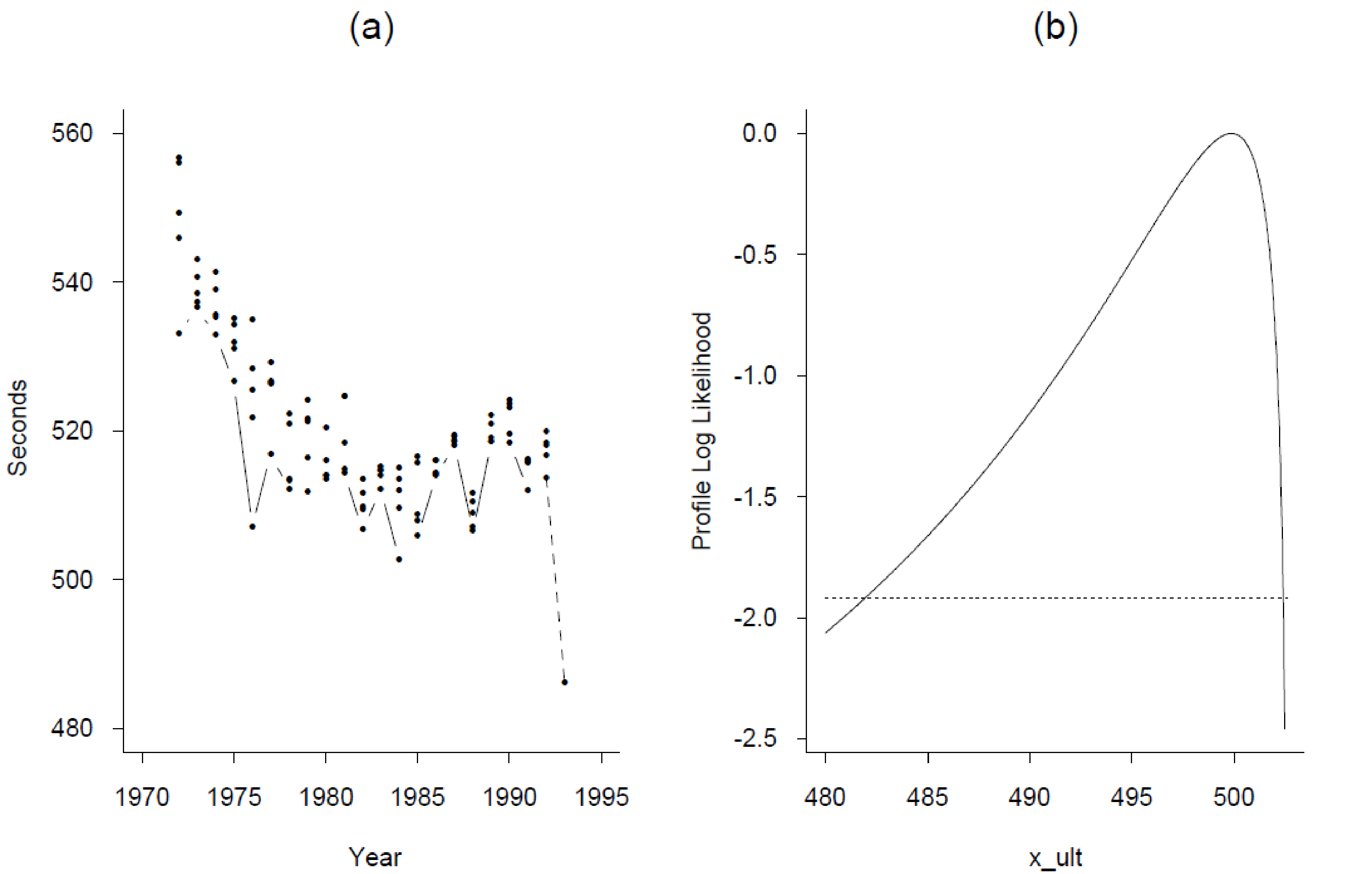
II.a Extreme value models

II.b An example based on track records



Plots of women's 3000 meter records, and profile log-likelihood for ultimate best value based on pre-1993 data.

Example. The left figure shows the five best running times by different athletes in the women's 3000 metre track event for each year from 1972 to 1992. Also shown on the plot is Wang Junxia's world record from 1993. Many questions were raised about possible illegal drug use.

We approach this by asking how implausible Wang's performance was, given all data up to 1992.

Robinson and Tawn (1995) used the r largest order statistics method (with $r = 5$, translated to smallest order statistics) to estimate an extreme value distribution, and hence computed a profile likelihood for x_{ult}, the lower endpoint of the distribution, based on data up to 1992 (right plot of previous figure)

Alternative Bayesian calculation:

(Smith 1997)

Compute the (Bayesian) predictive probability that the 1993 performance is equal or better to Wang's, given the data up to 1992, and conditional on the event that there is a new world record.

```
> yy=read.table('C:/Users/rls/r2/d/evt/marathon/w3000.txt',header=F)
> r=5
>
1  1972 533.00 545.80 549.20 556.00 556.60
2  1973 536.60 537.20 538.40 540.60 543.00
3  1974 532.80 535.20 535.60 539.00 541.40
4  1975 526.60 531.00 531.80 534.20 535.00
5  1976 507.12 521.80 525.40 528.40 534.90
6  1977 516.80 526.30 526.40 526.60 529.20
7  1978 512.10 513.20 513.50 520.90 522.30
8  1979 511.80 516.40 521.30 521.60 524.10
9  1980 513.53 513.90 514.00 516.00 520.40
10 1981 514.30 514.80 518.35 524.64 524.65
11 1982 506.78 509.36 509.71 511.67 513.40
12 1983 512.08 514.02 514.60 514.62 515.06
13 1984 502.62 509.59 512.00 513.57 514.91
14 1985 505.83 507.83 508.83 515.74 516.51
15 1986 513.99 514.10 514.43 515.92 516.00
16 1987 518.10 518.50 518.73 519.28 519.45
17 1988 506.53 507.15 509.02 510.45 511.67
18 1989 518.48 518.51 518.97 520.85 522.12
19 1990 518.38 519.46 523.14 523.68 524.07
20 1991 512.00 515.72 515.82 516.05 516.06
21 1992 513.72 516.63 517.92 518.45 519.94
```

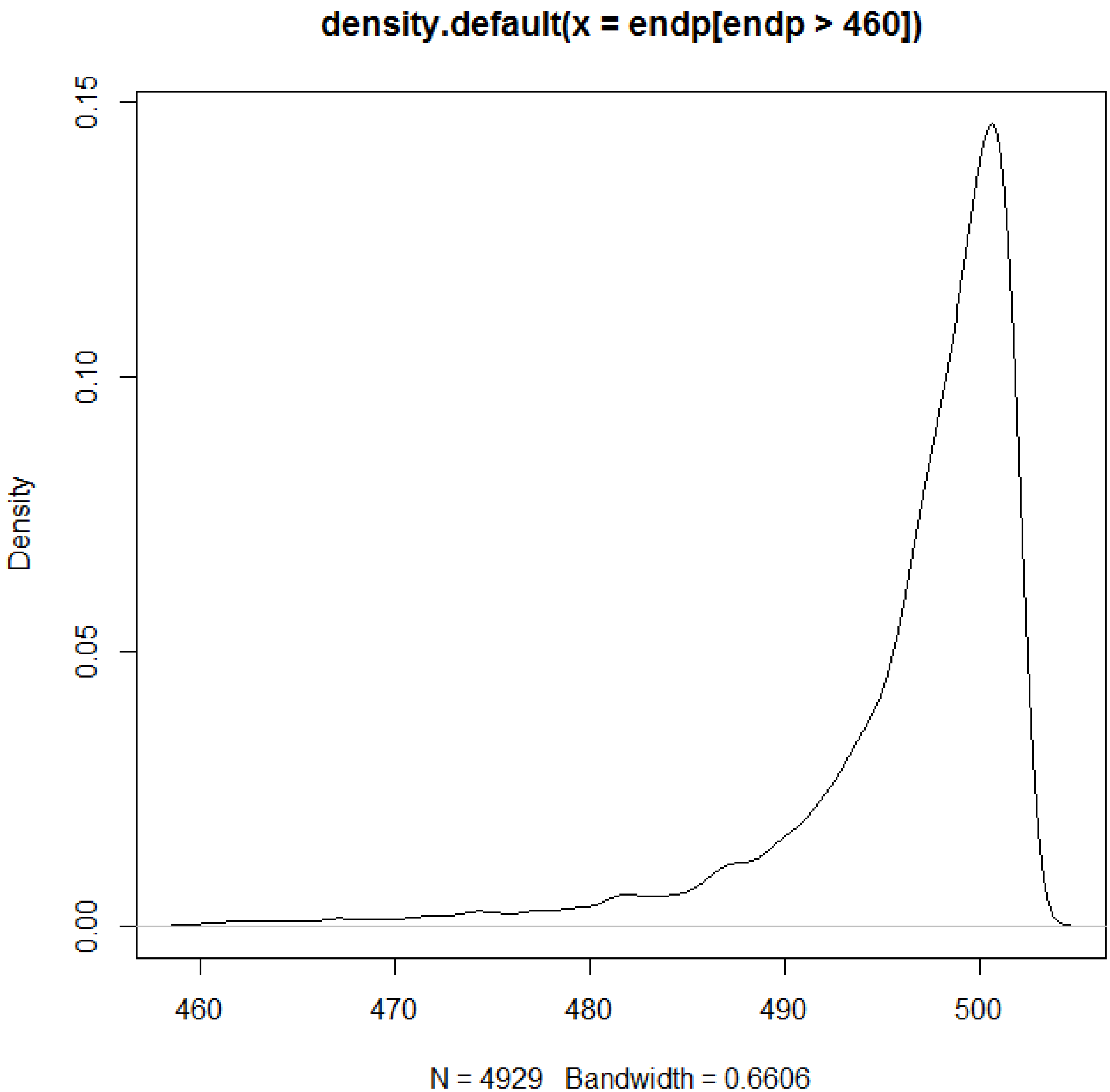
```
> # likelihood function (compute NLLH - defaults to 10^10 if parameter values
> #  infeasible) - par vector is (mu, log psi, xi)
> lh=function(par){
+ if(abs(par[2])>20){return(10^10)}
+ #if(abs(par[3])>1){return(10^10)}
+ if(par[3]>=0){return(10^10)}
+ mu=par[1]
+ psi=exp(par[2])
+ xi=par[3]
+ f=0
+ for(i in 9:21){
+ f=f+r*par[2]
+ s1=1+xi*(mu-yy[i,6])/psi
+ if(s1<=0){return(10^10)}
+ s1=-log(s1)/xi
+ if(abs(s1)>20){return(10^10)}
+ f=f+exp(s1)
+ for(j in 2:6){
+ s1=1+xi*(mu-yy[i,j])/psi
+ if(s1<=0){return(10^10)}
+ f=f+(1+1/xi)*log(s1)
+ }}
+ return(f)
+ }
```

```
> # trial optimization of likelihood function
> par=c(520,0,-0.01)
> lh(par)
[1] 485.5571
>
> par=c(510,1,-0.1)
> lh(par)
[1] 255.9864
>
> opt1=optim(par,lh,method="Nelder-Mead")
> opt2=optim(par,lh,method="BFGS")
> opt3=optim(par,lh,method="CG")
> opt1$par
[1] 510.8844846   1.3119151  -0.3377374
> opt2$par
[1] 510.8840970   1.3118407  -0.3378123
> opt3$par
[1] 510.4261195   1.3143073  -0.3549833
> opt1$value
[1] 116.1818
> opt2$value
[1] 116.1818
> opt3$value
[1] 116.3213
>
```

```
>
> # MLE of endpoint (intepreted as smallest possible running time)
>
> opt1$par[1]+exp(opt1$par[2])/opt1$par[3]
[1] 499.8899
> opt2$par[1]+exp(opt2$par[2])/opt2$par[3]
[1] 499.8928
>
> # now do more through optimization and prepare for MCMC
> par=c(520,0,-0.01)
> opt2=optim(par,lh,method="BFGS",hessian=T)
> library(MASS)
> A=ginv(opt2$hessian)
> sqrt(diag(A))
[1] 0.85637360 0.08829459 0.07802306
> eiv=eigen(A)
> V=eiv$vectors
> V=V %*% diag(sqrt(eiv$values)) %*% t(V)
```

```
> # MCMC - adjust nsim=total number of simulations,
> par=opt2$par
> nsim=1000000
> nsave=1
> nwrite=100
> del=1
> lh1=lh(par)
> parsim=matrix(nrow=nsim/nsave,ncol=3)
> accp=rep(0,nsim)
> for(isim in 1:nsim){
+ # Metropolis update step
+ parnew=par+del*V %*% rnorm(3)
+ lh2=lh(parnew)
+ if(runif(1)<exp(lh1-lh2)){
+ lh1=lh2
+ par=parnew
+ accp[isim]=1
+ }
+ if(nsave*round(isim/nsave)==isim){
+ parsim[isim/nsave,]=par
+ write(isim,'C:/Users/rls/mar11/conferences/NCSUFeb2015/counter.txt',ncol=1)
+ }
+ if(nwrite*round(isim/nwrite)==isim){
+ write(parsim,'C:/Users/rls/mar11/conferences/NCSUFeb2015/parsim.txt',ncol=1)}}
```

```
> # results from presaved MCMC output
> parsim1=matrix(scan('C:/Users/rls/mar11/conferences/NCSUFeb2015/parsim1.txt'),nr
Read 30000 items
> parsim=parsim1[(length(parsim1[,1])/2+1):length(parsim1[,1]),]
> s1=1+parsim[,3]*(parsim[,1]-502.62)/exp(parsim[,2])
> s1[s1<0]=s1
> s1[s1>0]=1-exp(-s1[s1>0]^(-1/parsim[s1>0,3]))
> s2=1+parsim[,3]*(parsim[,1]-486.11)/exp(parsim[,2])
> s2[s2<0]=0
> s2[s2>0]=1-exp(-s2[s2>0]^(-1/parsim[s2>0,3]))
> mean(s2/s1)
[1] 0.000422214
> mean(s2==0)
[1] 0.9212
> quantile(s2/s1,c(0.5,0.9,0.95,0.975,0.995))
         50%          90%          95%        97.5%        99.5%
0.0000000000 0.0000000000 0.0001011265 0.0026199509 0.0254551231
> endp=parsim[,1]+exp(parsim[,2])/parsim[,3]
> sum(endp<486.11)/length(endp)
[1] 0.079
> plot(density(endp[endp>460]))
```


density.default(x = endp[endp > 460])
Density
0.00
0.05
0.10
0.15
460
470
480
490
500
N = 4929 Bandwidth = 0.6606

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II.a Extreme value models

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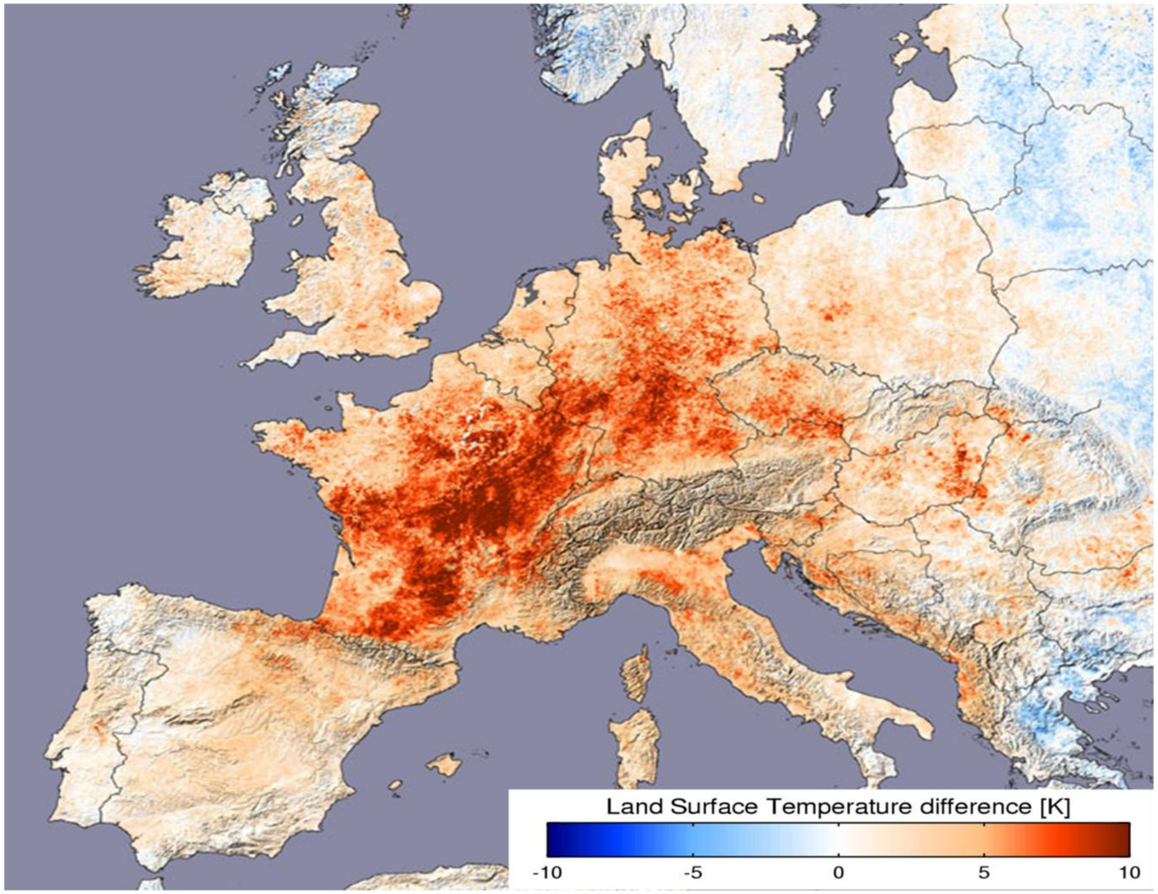
II.c Applying extreme value models to weather extremes



European temperatures in early August 2003, relative to 2001-2004 average

From NASA's MODIS - Moderate Resolution Imaging Spectrometer, courtesy of Reto Stöckli, ETHZ

Motivating Question:

- Concern over increasing frequency of extreme meteorological events
  - Is the increasing frequency a result of anthropogenic influence?
  - How much more rapidly with they increase in the future?

- Focus on three specific events: heatwaves in Europe 2003, Russia 2010 and Central USA 2011

- Identify meteorological variables of interest — JJA temperature averages over a region
  - Europe — 10^o W to 40^o E, 30^o to 50^o N
  - Russia — 30^o to 60^o E, 45^o to 65^o N
  - Central USA — 90^o to 105^o W, 25^o to 45^o N

- Probabilities of crossing thresholds — respectively 1.92K, 3.65K, 2.01K — in any year from 1990 to 2040.

Data

Climate model runs have been downloaded from the WCRP CMIP3 Multi-Model Data website (http://esg.llnl.gov:8080/index.jsp)

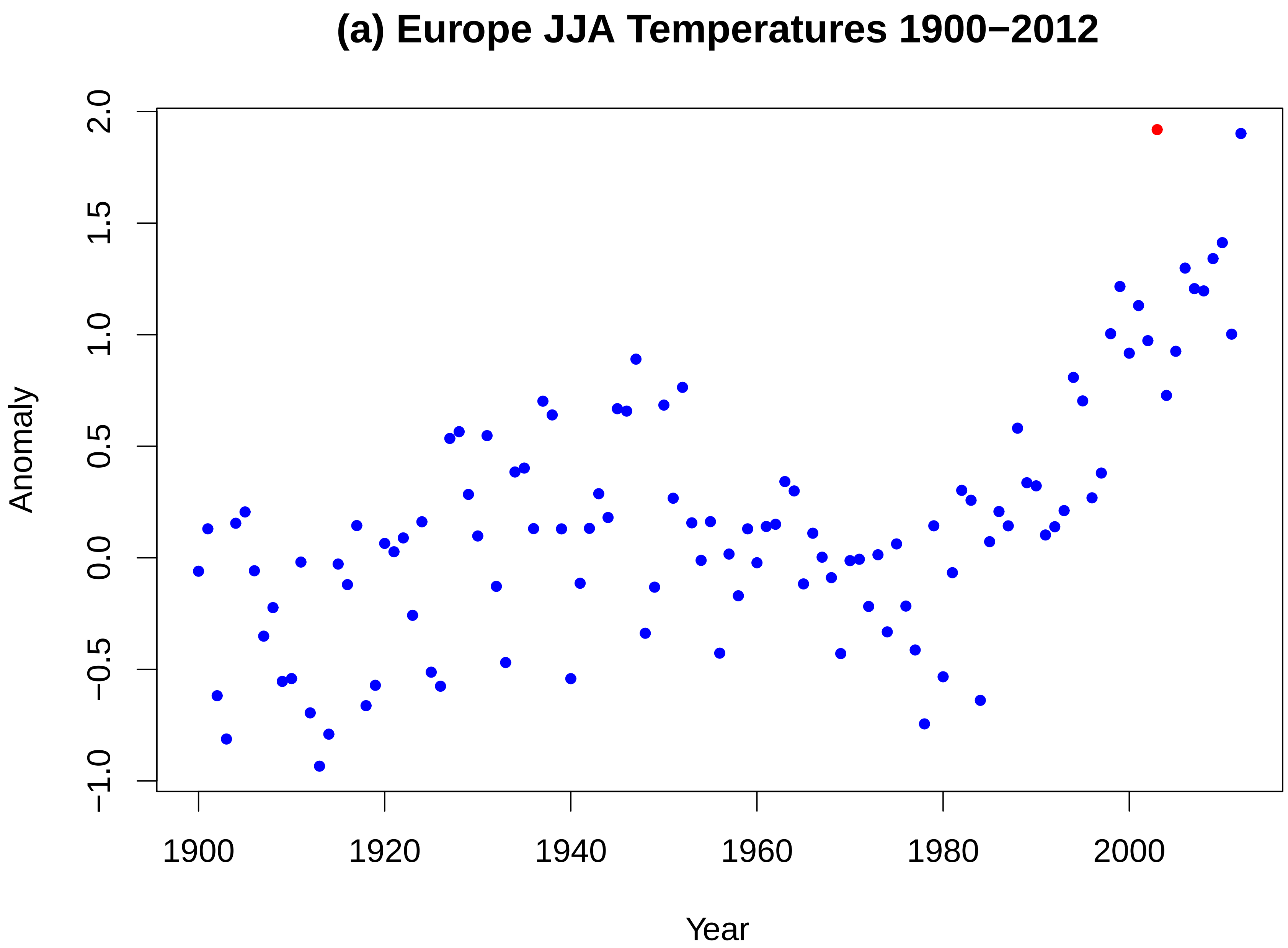
Three kinds of model runs:

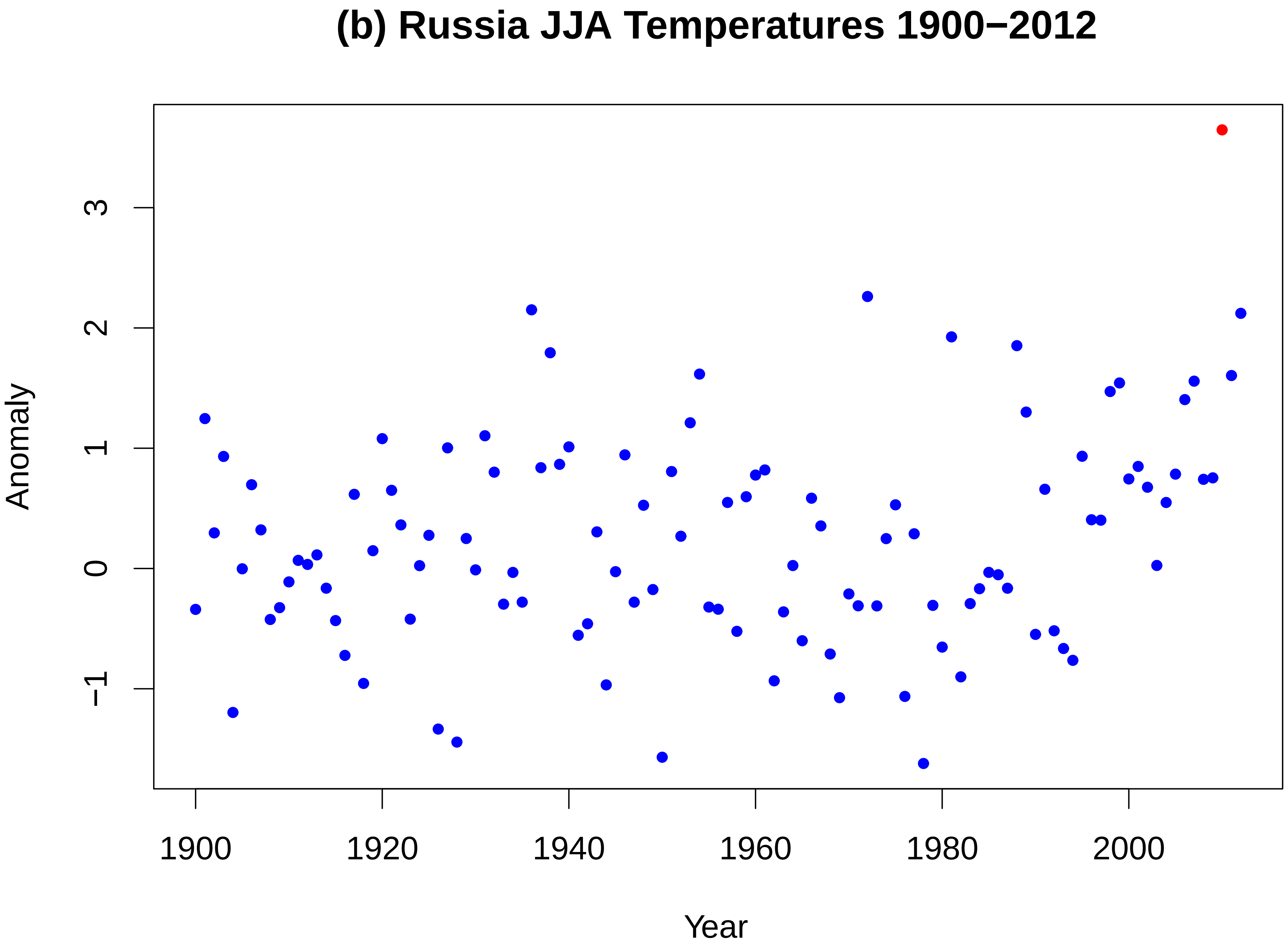
- Twentieth-century
- Pre-industrial control model runs (used a proxy for natural forcing)
- Future projections (A2 scenario)

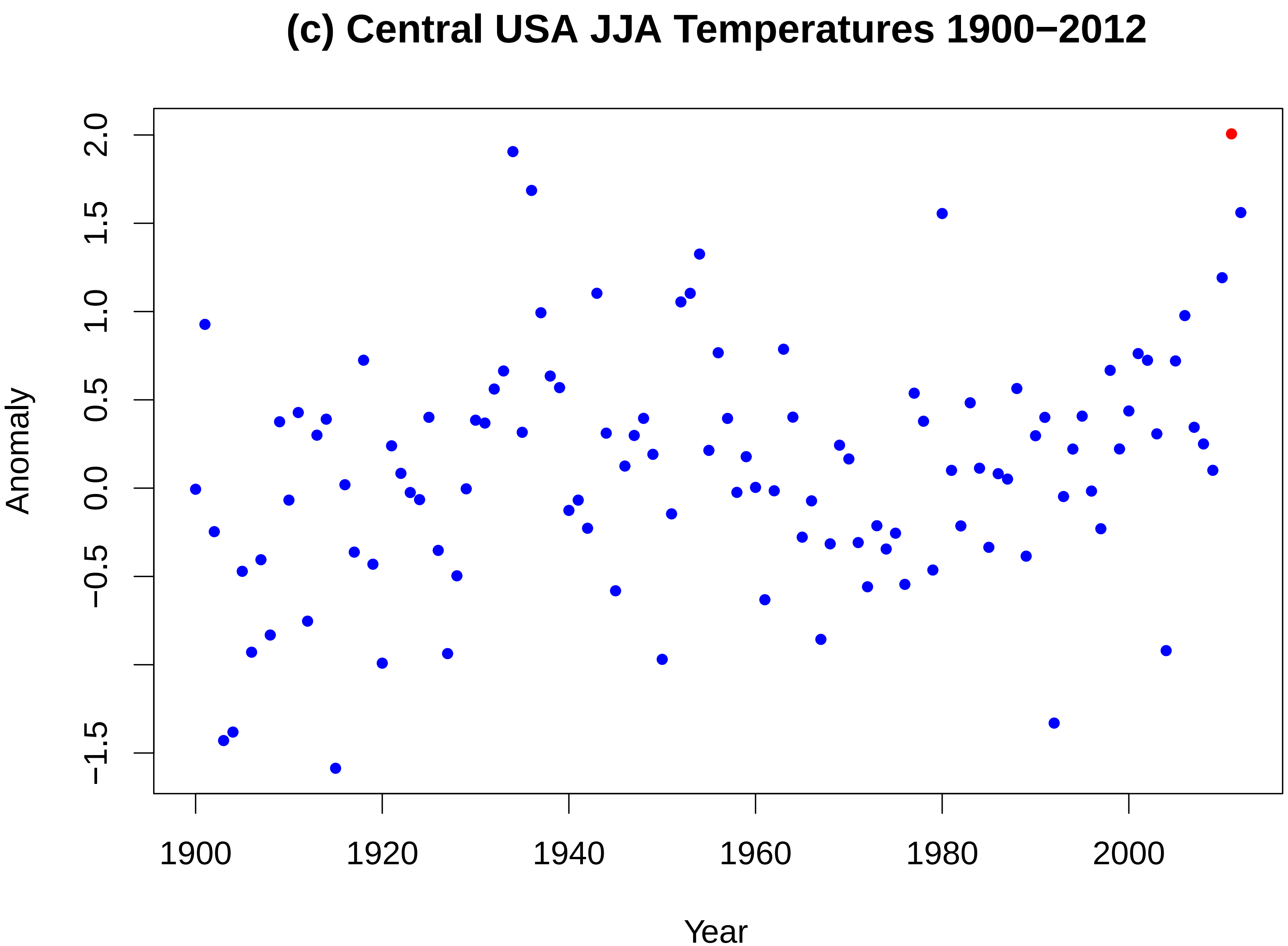
We also took observational data ($5^o \times 5^o$ gridded monthly temperature anomalies) from the website of the Climate Research Unit of the University of East Anglia (www.cru.uea.ac.uk — HadCRUT3v dataset)

Number	Model	Control runs	20C runs	A2 runs
1	bccr_bcm2_0	2	1	1
2	cccma_cgcm3_1	10	5	5
3	cnrm_cm3	5	1	1
4	csiro_mk3_0	3	3	1
5	gfdl_cm2_1	5	3	1
6	giss_model_e_r	5	9	1
7	ingv_echam4	1	1	1
8	inmcm3_0	3	1	1
9	ipsl_cm4	7	1	1
10	miroc3_2_medres	5	3	3
11	mpi_echam5	5	4	3
12	mri_cgcm2_3_2a	3	5	5
13	ncar_ccsm3_0	7	5	5
14	ukmo_hadcm3	3	2	1

List of climate models, including numbers of runs available under three scenarios


(a) Europe JJA Temperatures 1900−2012
Anomaly
Year
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
1900
1920
1940
1960
1980
2000


(b) Russia JJA Temperatures 1900−2012
Anomaly
−1
0
1
2
3
1900
1920
1940
1960
1980
2000
Year


(c) Central USA JJA Temperatures 1900−2012
Anomaly
2.0
1.5
1.0
0.5
0.0
−0.5
−1.5
1900
1920
1940
1960
1980
2000
Year

Analysis of Observational Data

Key tool: *Generalized Extreme Value* Distribution (GEV)

- Three-parameter distribution, derived as the general form of limiting distribution for extreme values (Fisher-Tippett 1928, Gnedenko 1943)

- μ, σ, ξ known as location, scale and shape parameters

- $\xi > 0$ represents long-tailed distribution, $\xi < 0$ short-tailed

Formula:

$$\text{Pr}\{Y \leq y\} = \exp\left[-\left\{1 + \xi\left(\frac{y-\mu}{\sigma}\right)\right\}_+^{-1/\xi}\right].$$

- *Peaks over threshold* approach implies that the GEV can be used generally to study the tail of a distribution: assume GEV holds exactly above a threshold u and that values below u are treated as left-censored

- Time trends by allowing μ, σ, ξ to depend on time

- *Example:* Allow $\mu_t = \beta_0 + \sum_{k=1}^{K} \beta_k x_{kt}$ where $\{x_{kt},\ k = 1, ..., K,\ t = 1, ..., T\}$ are spline basis functions for the approximation of a smooth trend from time 1 to T with K degrees of freedom

- Critical questions:
  - Determination of threshold and K
  - Estimating the probability of exceeding a high value such as 1.92K

Application to Temperature Series

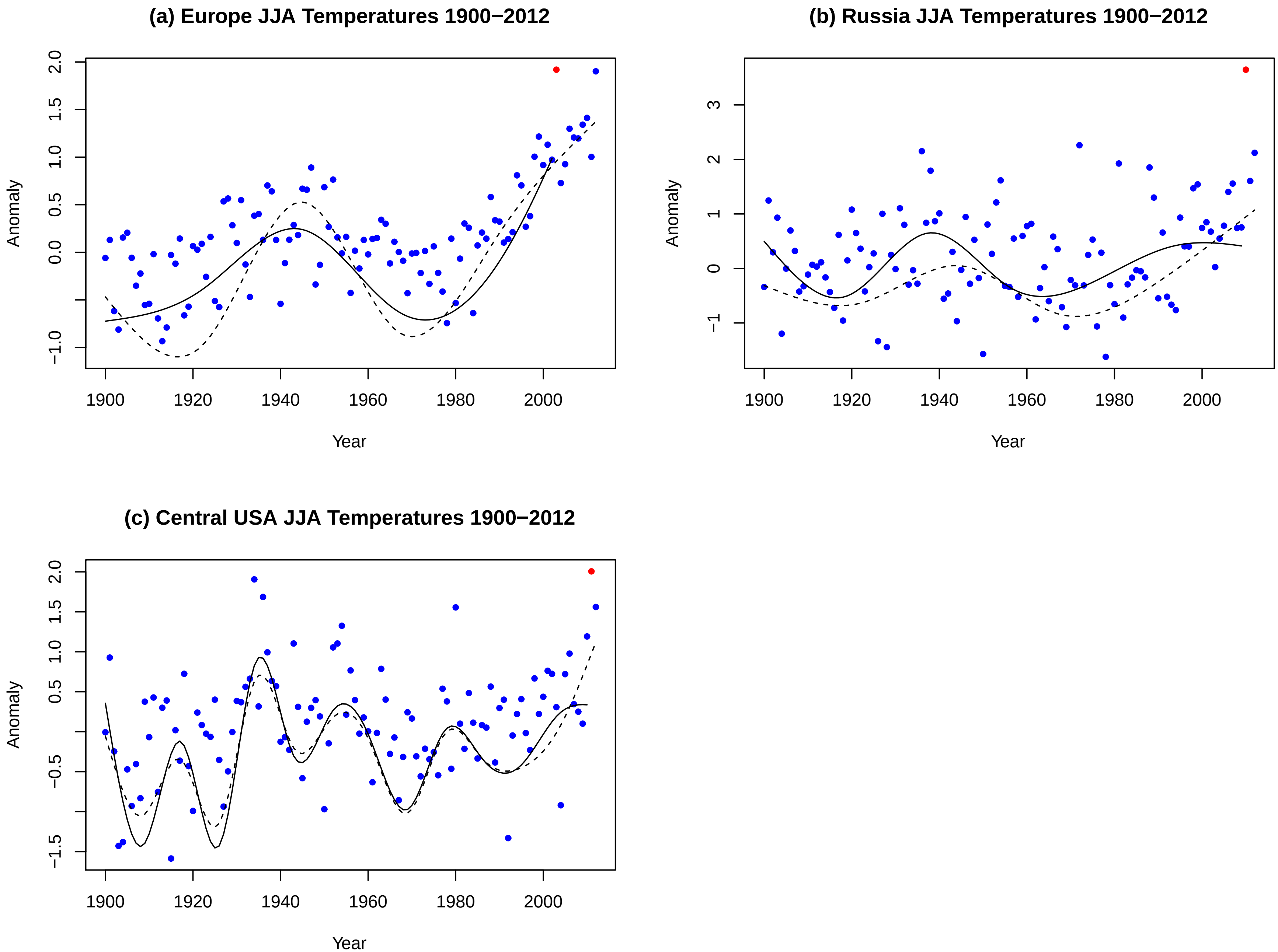
- GEV with trend fitted to three observational time series
- Threshold was chosen as fixed quantile — 75th, 80th or 85th percentile
- AIC was used to help select the number of spline basis terms K
- Estimate probability of extreme event by maximum likelihood (MLE) or Bayesian method
- Repeat the same calculation with no spline terms
- Use full series or part?
- Examine sensitivity to threshold choice through plots of the posterior densities.

K Threshold	Europe 75%	80%	85%	Russia 0.75	0.8	0.85	Texas 0.75	0.8	0.85
2	97.9	87.7	67.5	149.8	131.2	110.4	146.6	131.3	108.8
3	75.7	68.5	60.5	145.8	135.4	112.7	142.6	125.0	105.5
4	76.1	66.2	44.9	148.1	137.8	113.8	144.6	126.8	103.6
5	74.1	64.6	54.6	147.0	134.1	121.2	144.1	126.5	104.9
6	74.2	74.3	61.6	146.8	133.6	113.1	143.8	125.5	106.1
7	77.9	75.2	59.8	146.6	135.1	114.0	133.4	126.4	106.8
8	86.2	77.4	65.9	148.0	137.1	122.1	138.9	128.4	108.1
9	86.8	74.6	67.1	149.4	138.7	113.3	148.6	130.6	110.2
10	88.7	94.8	54.2	150.8	140.4	125.1	128.2	122.9	105.7
11	90.6	73.4	73.5	153.1	142.6	125.7	144.2	127.8	110.5
12	79.1	98.6	59.3	152.8	140.8	126.4	135.1	119.7	105.8
13	95.3	79.6	59.1	156.1	144.2	127.4	136.2	116.9	104.2
14	77.5	78.6	54.6	157.5	142.4	128.7	138.9	121.8	107.9
15	97.6	85.5	77.9	157.2	143.1	129.5	136.8	122.5	109.6

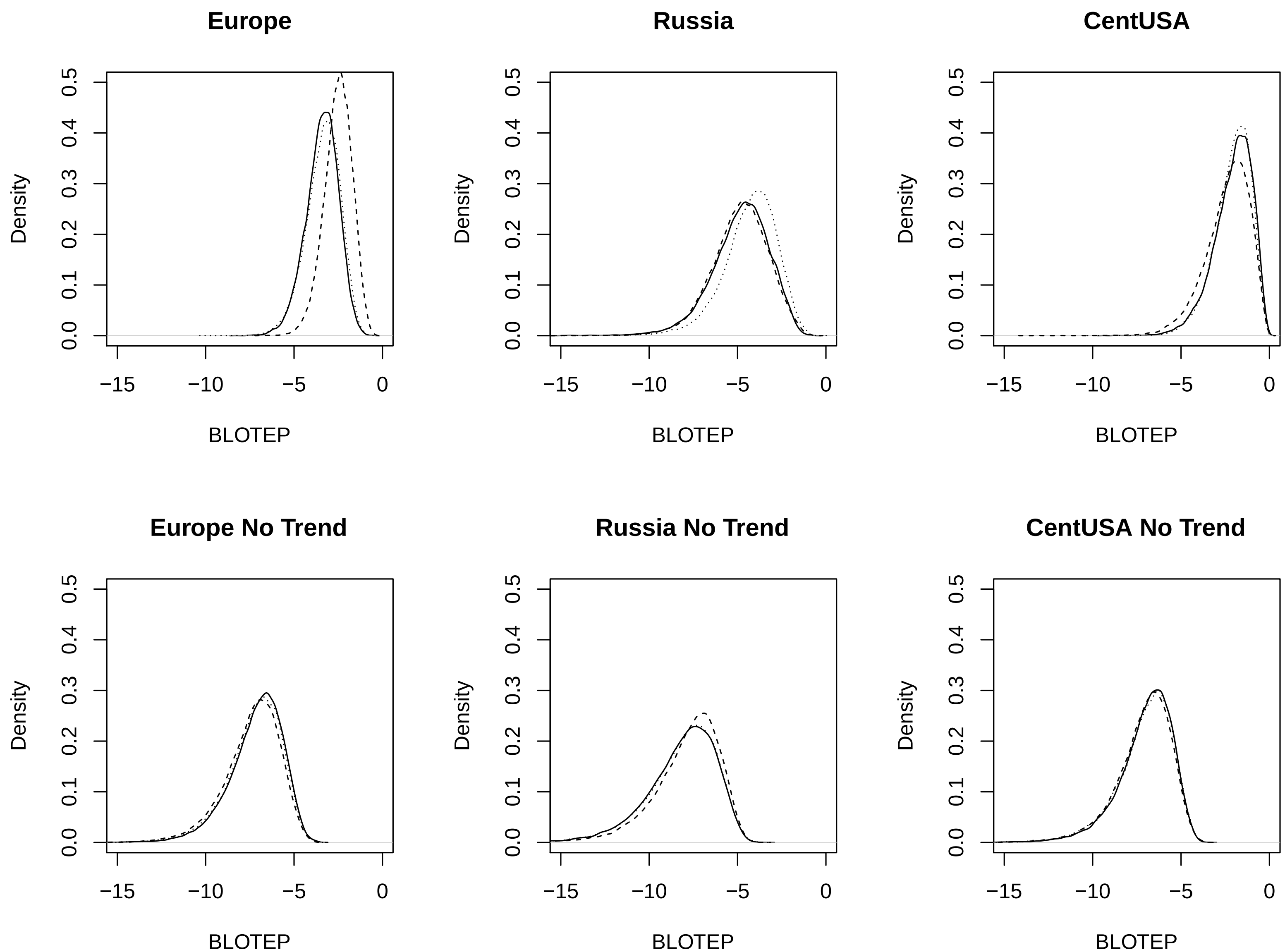
AIC values for different values of K, at three different thresholds, for each dataset of interest. In each column, the smallest three AIC values are indicated in red, green and blue respectively.

Dataset	Endpoint	K	Threshold	MLE	Posterior Mean	Posterior Quantiles 0.05	0.5	0.95
Europe	2002	5	80%	.021	.076	0	.057	.217
Europe	2012	5	80%	.0027	.113	.031	.098	.246
Europe	2002	0	80%	0	.0004	0	0	.002
Europe	2012	0	80%	.0044	.011	.001	.0081	.029
Russia	2009	6	80%	.0013	.010	0	.004	.040
Russia	2012	5	80%	.010	.058	.005	.039	.181
Russia	2009	0	80%	0	.0011	0	0	.0069
Russia	2012	0	80%	.0019	.0067	.0003	.0043	.021
CentUSA	2010	13	80%	.0007	.072	.003	.045	.234
CentUSA	2012	13	80%	.089	.300	.058	.268	.653
CentUSA	2010	0	80%	.0023	.0078	.00007	.0052	.024
CentUSA	2012	0	80%	.005	.012	.001	.0092	.031

Results of extreme value analysis applied to observational datasets. For three datasets (Europe, Russia, Central USA), different choices of the endpoint of the analysis, spline degrees of freedom K, and threshold, we show the maximum likelihood estimate (MLE) of the probability of the extreme event of interest, as well as the posterior mean and three quantiles of the posterior distribution.



Plot of three time series for 1900–2012, with fitted trend curves.

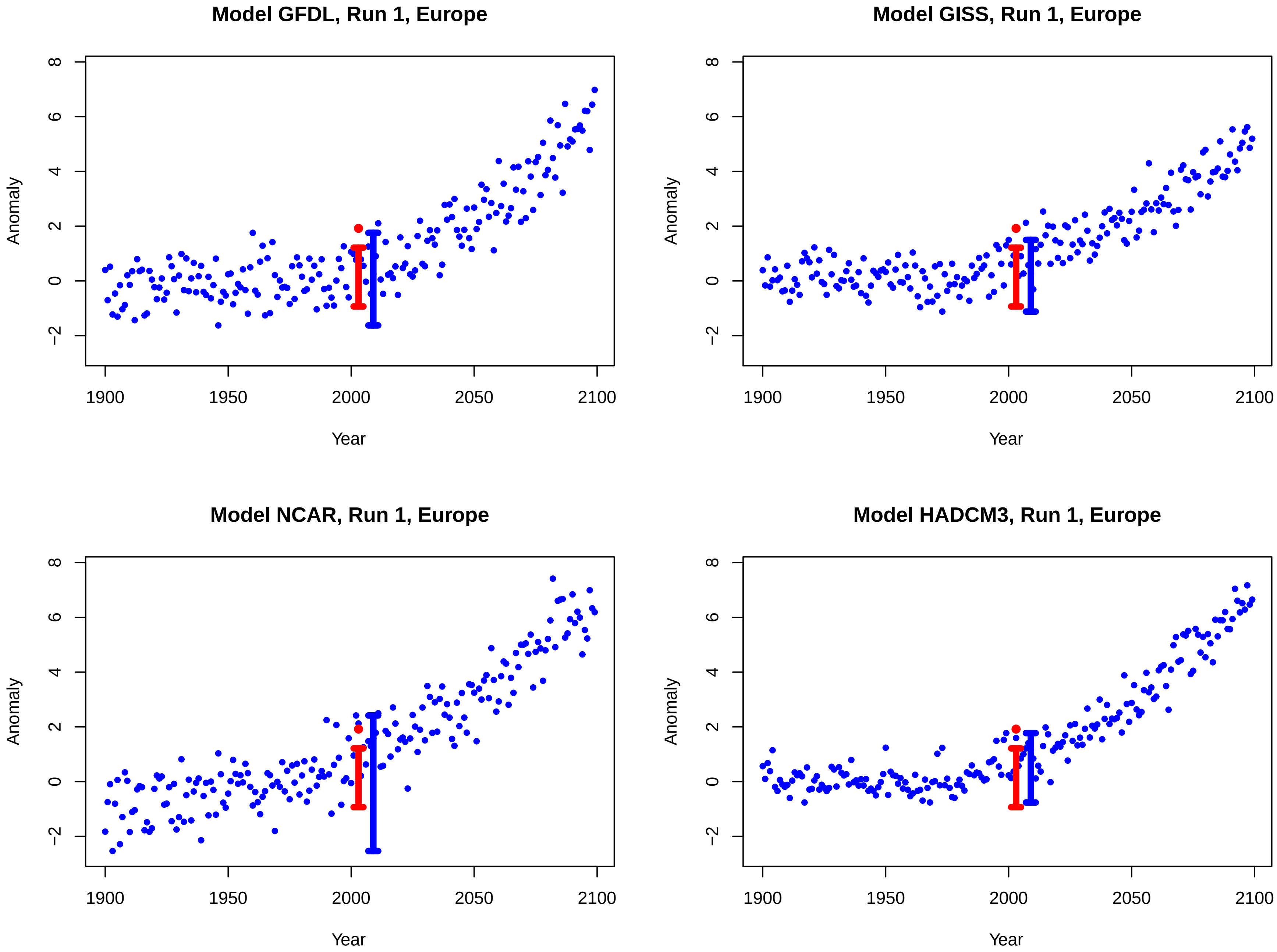


Posterior densities of the BLOTEP, with (top) and without (bottom) spline-based trends. Based on 80% (solid curve), 75% (dashed) and 85% (dot-dashed) thresholds.

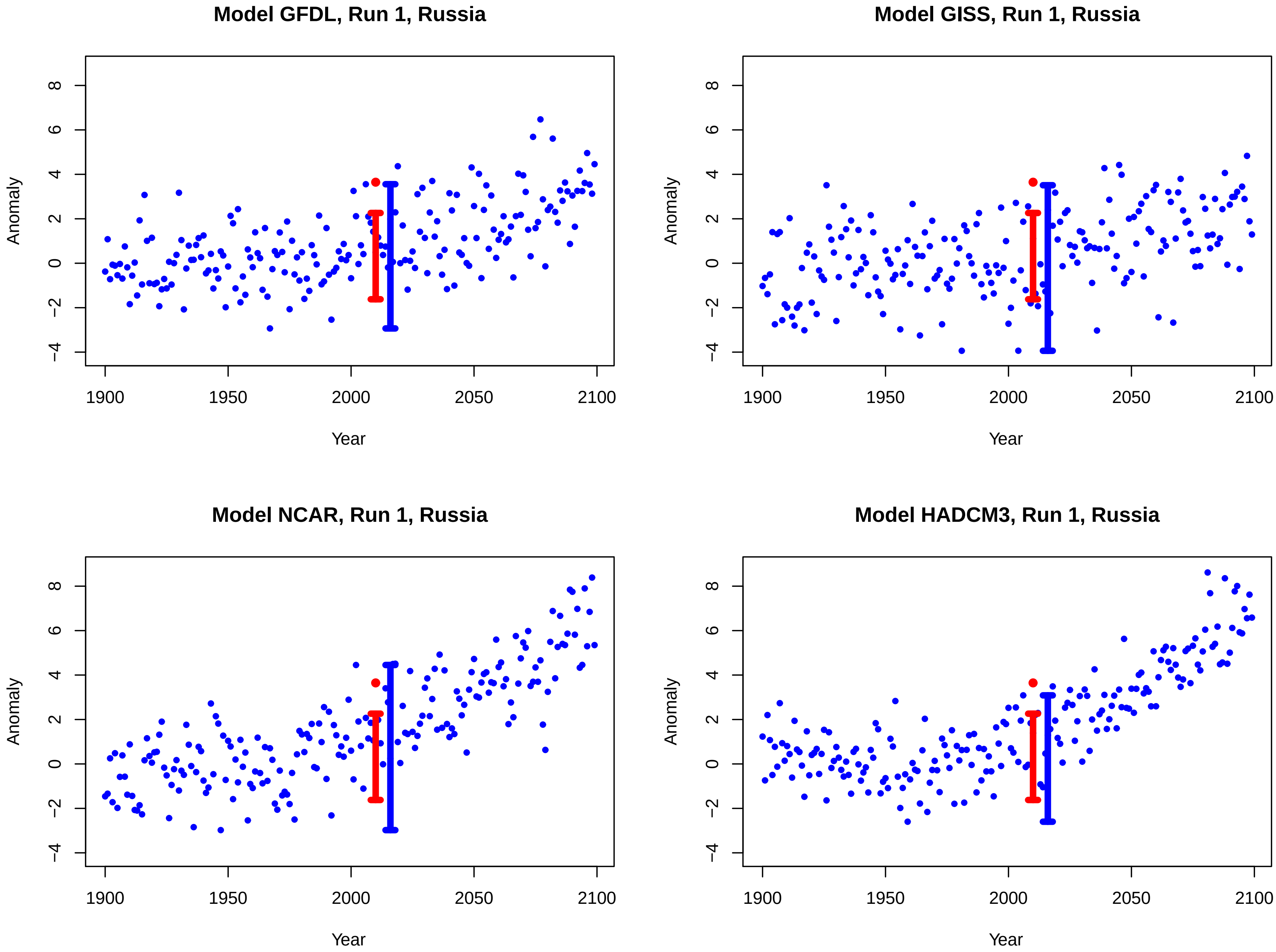
Summary So Far:

- Estimate extreme event probabilities by GEV with trends
- Bayesian posterior densities best way to describe uncertainty
- Two major disadvantages:
  - No way to distinguish anthropogenic climate change effects from other short-term fluctations in the climate (El Niños and other circulation-based events; the 1930s dust-bowl in the US)
  - No basis for projecting into the future

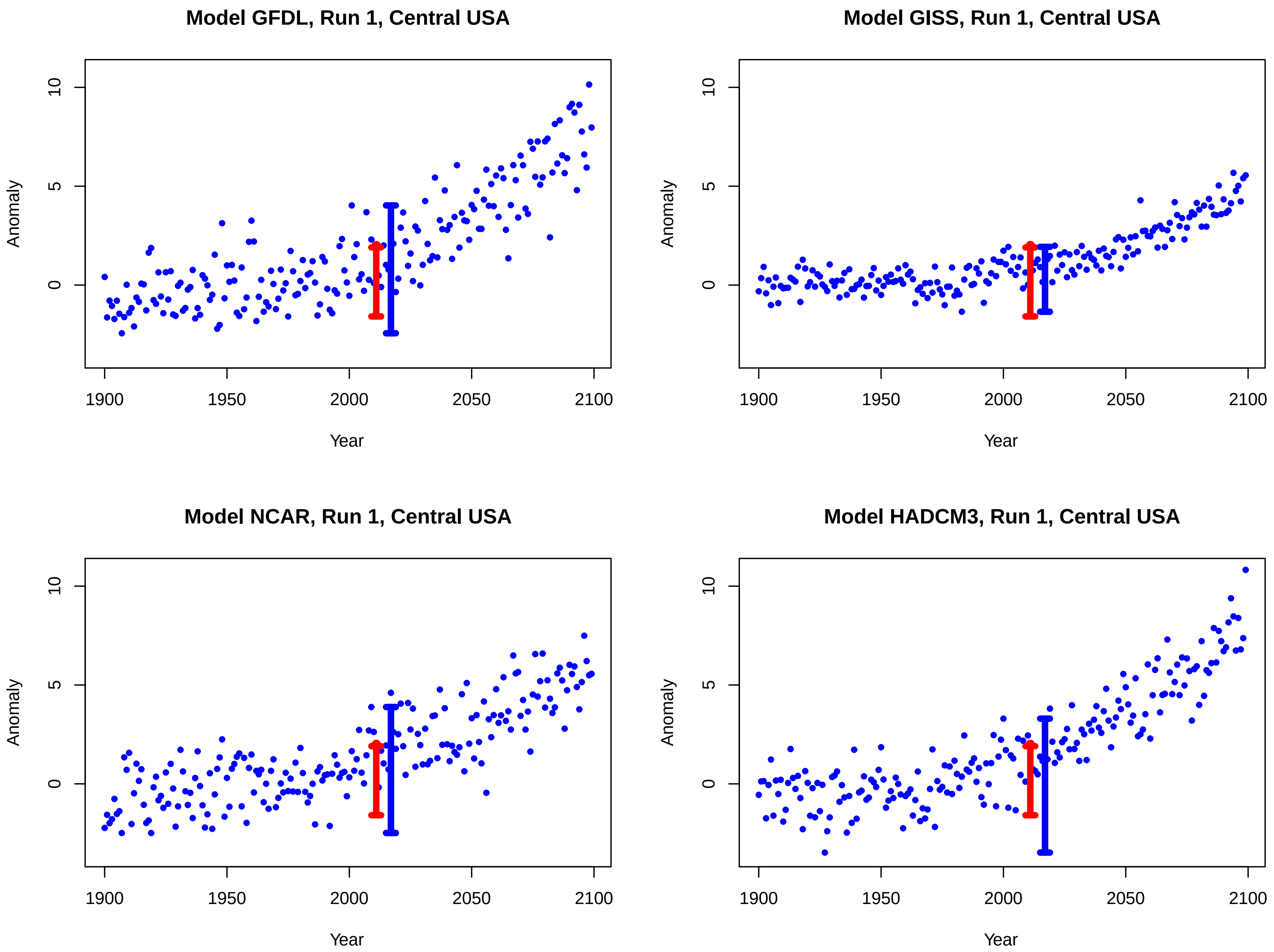
It might seem that the way to do future projections is simply to rerun the analysis based on climate model data instead of observations. However, this runs into the *scale mismatch* problem.



Scale mismatch: 4 model runs (range of observations in red).



Scale mismatch: 4 model runs (range of observations in red).



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II.d Joint distributions of two of more variables

Example 1. Herweijer and Seager (2008) argued that the persistence of drought patterns in various parts of the world may be explained in terms of SST patterns. One of their examples (Figure 3 of their paper) demonstrated that precipitation patterns in the south-west USA are highly correlated with those of a region of South America including parts of Uruguay and Argentina.

I computed annual precipitation means for the same regions, that show the two variables are clearly correlated (r=0.38; p¡.0001). The correlation coefficient is lower than that stated by Herweijer and Seager (r=0.57) but this is explained by their use of 6-year moving average filter, which naturally increases the correlation.

Our interest here: look at dependence in lower tail probabilities

Transform to unit Fréchet distribution (small values of precipitation corresponding to large values on Frchet scale)

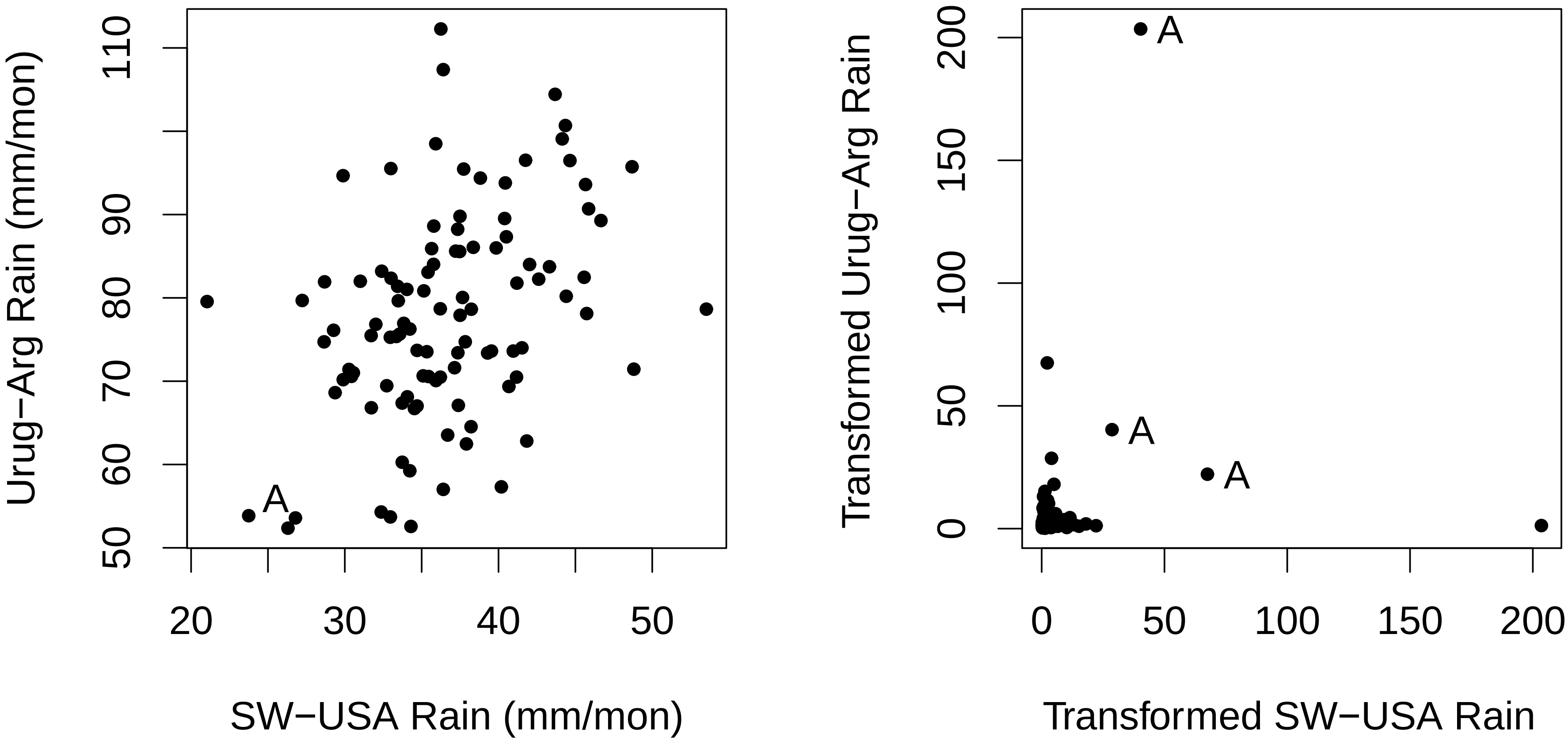

Figure 1. Left: Plot of USA annual precipitation means over latitudes 25-35°N, longitudes 95-120°W, against Argentina annual precipitation means over latitudes 30-40°S, longitudes 50-65°W, 1901-2002. Right: Same data with empirical transformation to unit Fréchet distribution. Data from gridded monthly precipitation means archived by the Climate Research Unit of the University of East Anglia.

Example 2. Lau and Kim (2012) have provided evidence that the 2010 Russian heatwave and the 2010 Pakistan floods were derived from a common set of meteorological conditions, implying a physical dependence between these very extreme events.

Using the same data source as for Example 1, I have constructed summer temperature means over Russia and precipitation means over Pakistan corresponding to the spatial areas used by Lau and Kim.

Scatterpolt of raw data and unit Fréchet transformation. 2010 value approximated — an outlier for temperature but not for precipitation.

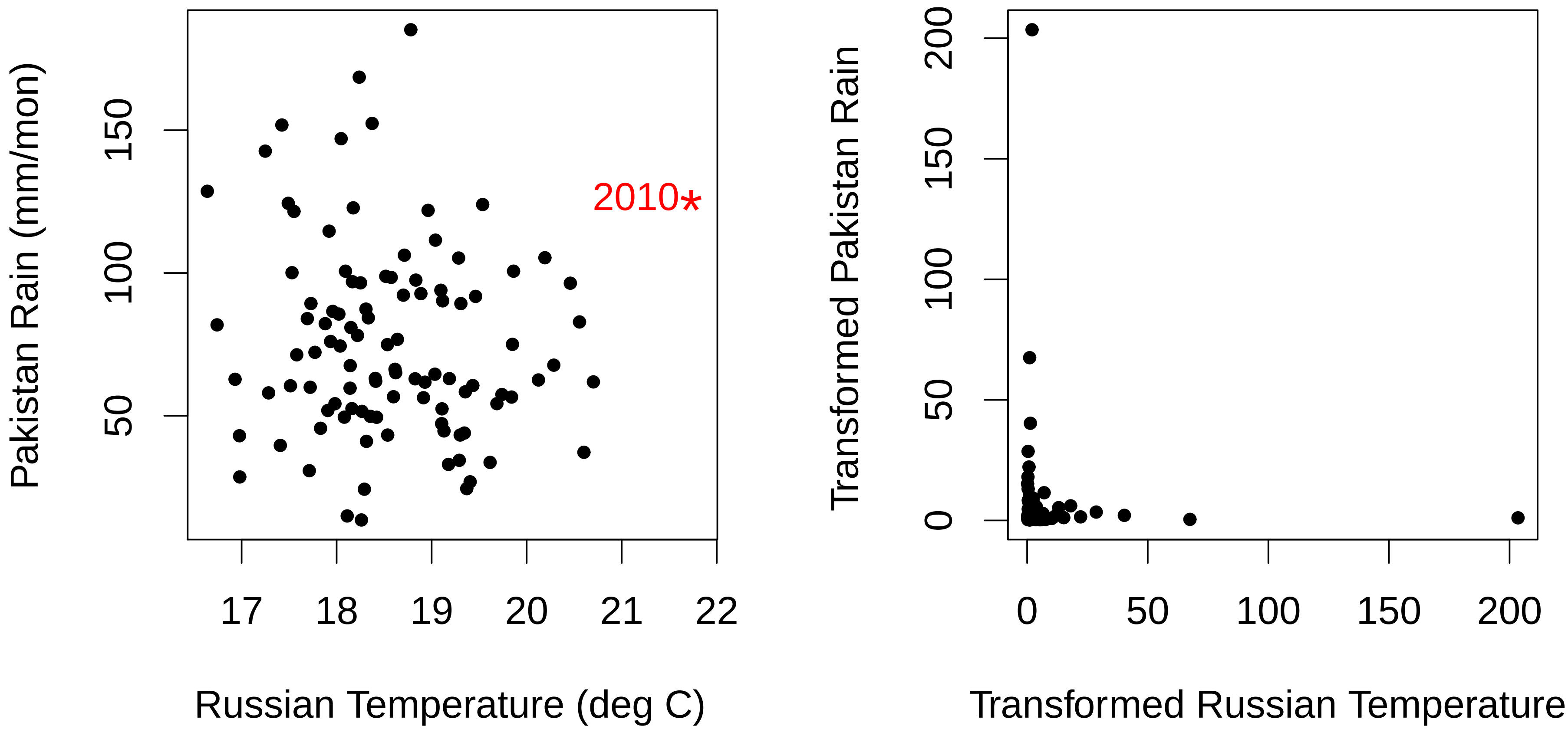

Figure 2. Left: Plot of JJA Russian temperature means against Pakistan JJA precipitation means, 1901-2002. Right: Same data with empirical transformation to unit Fréchet distribution. Data from CRU, as in Figure 1. The Russian data were averaged over 45-65°N, 30-60°E, while the Pakistan data were averaged over 32-35°N, 70-73°E, same as in Lau and Kim (2012).

Methods

Focus on the *proportion* by which the probability of a joint exceedance is greater than what would be true under independence.

Method: Fit a joint bivariate model to the exceedances above a threshold on the unit Fréchet scale

Two models:

- Classical logistic dependence model (Gumbel and Mustafi 1967; Coles and Tawn 1991)
- The η-asymmetric logistic model (Ramos and Ledford 2009)

	Logistic Model		Ramos-Ledford Model	
	Estimate	90% CI	Estimate	90% CI
10-year	2.7	(1.2 , 4.2)	2.9	(1.2 , 5.0)
20-year	4.7	(1.4 , 7.8)	4.9	(1.2 , 9.6)
50-year	10.8	(2.1 , 18.8)	9.9	(1.4 , 23.4)

Table 1. Estimates of the increase in probability of a joint extreme event in both variables, relative to the probability under independence, for the USA/Uruguay-Argentina precipitation data. Shown are the point estimate and 90% confidence interval, under both the logistic model and the Ramos-Ledford model.

	Logistic Model		Ramos-Ledford Model	
	Estimate	90% CI	Estimate	90% CI
10-year	1.01	(1.00 , 1.01)	0.33	(0.04 , 1.4)
20-year	1.02	(1.00 , 1.03)	0.21	(0.008 , 1.8)
50-year	1.05	(1.01 , 1.07)	0.17	(0.001 , 2.9)

Table 2. Similar to Table 1, but for the Russia-Pakistan dataset.

Conclusions

- The USA–Argentina precipitation example shows clear dependence in the lower tail, though the evidence for that rests primarily on three years' data

- In contrast, the analysis of Russian temperatures and Pakistan rainfall patterns shows no historical correlation between those two variables

- Implications for future analyses: the analyses also show the merits of the Ramos-Ledford approach to bivariate extreme value modeling. The existence of a parametric family which is tractable for likelihood evaluation creates the possibility of constructing hiterarchical models for these problems.

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II.e Conclusions, other models, future research

At least three methodological extensions, all of which are topics of active research:

1. Models for multivariate extremes in > 2 dimensions

2. Spatial extremes: max-stable process, different estimation methods
   (a) Composite likelihood method
   (b) Open-faced sandwich approach
   (c) Approximations to exact likelihood, e.g. ABC method

3. Hierarchical models for bivariate and spatial extremes?